

IIST

IIT FOUNDATION

Integrated Syllabus

TEACHER'S HAND BOOK



MATHEMATICS

CLASS IX

INDIAN INSTITUTE OF SOFTWARE TECHNOLOGY

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EXERCISE - 1**POLYNOMIALS****I. Multiple Choice Questions.**

1. If one root of the polynomial $5x^2 + 13x + k$ is reciprocal of the other, then the value of k is:
 a) 0 b) 5 c) $1/6$ d) None of these

Sol: Let $p(x) = 5x^2 + 13x + k$

$$\alpha = \frac{1}{\beta} \Rightarrow \alpha \cdot \beta = 1$$

$$\text{But from the polynomial } \alpha \cdot \beta = \frac{k}{5}$$

$$\therefore \frac{k}{5} = 1 \quad \text{so } k = 5 \quad \text{Ans: (B) 5}$$

2. If the product of two zeros of the polynomial $2x^3 + 6x^2 - 4x + 9$ is 3, then its third zero is:
 a) $3/2$ b) $-3/2$ c) $9/2$ d) none of these

Sol: $p(x) = 2x^3 + 6x^2 - 4x + 9$

$$\alpha \cdot \beta = 3$$

$$\alpha \cdot \beta \cdot \gamma = \frac{-d}{a} = \frac{-9}{2}$$

$$\Rightarrow 3\gamma = \frac{-9}{2}$$

$$\therefore \gamma = \frac{-3}{2} \quad \text{Ans: (B) } = -3/2$$

3. The number of real zeros of the polynomial $x^4 + 4x^2 + 5$ is:
 a) one b) two c) three d) None of these

Ans: (D) None of these

4. If the polynomials $2x^3 + 9x^2 + 3x - 5$ and $x^3 + x^2 - 4x + a$ leave the same remainder when divided by $x - 2$ then the value of a is:

- a) $\frac{3}{13}$ b) $\frac{3}{17}$ c) $-\frac{3}{13}$ d) None of these

Sol: $p(x) = 2x^3 + 9x^2 + 3x - 5$

$$q(x) = x^3 + x^2 - 4x + a$$

$$p(2) = 2 \times 2^3 + 9 \times 2^2 + 3 \times 2 - 5 = 17 + 4a$$

$$q(2) = 2^3 + 2^2 - 4 \times 2 + a = 4 + a$$

$$\therefore 4 + a = 17 + 4a \Rightarrow a = \frac{-13}{3} \quad \text{Ans: (C) } -\frac{3}{13}$$

5. When a polynomial $2x^3 + 5x^2 - 2x - 6$ is divided by $x + 1$ then the remainder will be:
 a) 1 b) -1 c) 2 d) None of these

Sol: $p(x) = 2x^3 + 5x^2 - 2x - 6$

$P(-1) = 2 \times (-1)^3 + 5 \times (-1)^2 - 2 \times (-1) - 6 = -1$ **Ans: (B) -1**

6. The minimum value of expression $x^2 + x - 2$ is :

- a) $\frac{9}{4}$ b) $-\frac{9}{4}$ c) $\frac{3}{4}$ d) None of these

Sol: $p(x) = x^2 + x - 2$

The minimum value of the expression = $\frac{-D}{4a} = \frac{-(b^2 - 4ac)}{4a} = \frac{-(2^2 - 4 \times 4 \times 1)}{4 \times 4} = \frac{3}{4}$

Ans: (C) $\frac{3}{4}$

7. If $x + \frac{1}{x} = 3$ then $x^2 + \frac{1}{x^2} =$ _____

- a) 9 b) 11 c) 7 d) None of these

Sol: $x + \frac{1}{x} = 3$ (Squaring both sides)

$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 3^2$

$\Rightarrow x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x} = 9$

$\Rightarrow x^2 + \frac{1}{x^2} = 9 - 2 = 7$ **Ans: (C) 7**

8. If $a + b = 10$ and $ab = 21$ then $a^2 + b^2 =$ _____

- a) 37 b) 36 c) 58 d) None of these

Sol: $a + b = 10$, $a.b = 21$

$a^2 + b^2 = (a + b)^2 - 2ab = 10^2 - 2 \times 21 = 58$ **Ans: (C) 58**

9. The value of k for which $x + k$ is a factor of $x^3 + kx^2 - 2x + k + 4$ is _____.

- a) -5 b) 2 c) $\frac{6}{7}$ d) None of these

Sol: The value of k for which $x + k$ is a factor of $x^3 + kx^2 - 2x + k + 4$

$f(-k) = (-k)^3 + k(-k)^2 - 2(-k) + k + 4 = 0$

$\Rightarrow 3k + 4 = 0$

$\Rightarrow 3k = -4$

$\Rightarrow k = \frac{-4}{3}$

Ans: (D) None of these

10. $x^2 + \frac{1}{x^2} = 62$ then the value of $x^4 + \frac{1}{x^4} =$ _____

a) 3842

b) 3840

c) 3838

d) None of these

Sol: $x^2 + \frac{1}{x^2} = 62$ (Squaring both side)

$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 62^2 \quad [(a+b)^2 = a^2 + b^2 + 2ab]$$

$$\Rightarrow x^4 + \frac{1}{x^4} + 2 \times x^2 + \frac{1}{x^2} = 3844 \Rightarrow x^4 + \frac{1}{x^4} = 3844 - 2 = 3842$$

Ans: (A) 3842

II. Say True or False.

1. $3x^3 - 5x + 6$ is a polynomial of degree 2. [False]
2. Every polynomial is a polynomial of degree 2. [False]
3. Every polynomial is a trianomial. [False]
4. A polynomial may have more than one zeros. [True]
5. The number of zeros of a polynomial is the degree of the polynomial. [True]
6. The graph of the polynomial $x^2 - 4$ crosses the x axis in 2 points. [True]
7. The zeros of the polynomial $x^2 - 1$ are 2 and -2. [False]
8. When $x^3 - 1$ is divided by $x - 1$ the remainder is 1. [False]
9. When $x^3 + 1$ is divided by $x + 1$ the remainder is 0. [True]
10. The sum of zeros of the polynomial $x^2 - 1$ is 1. [False]

III. Match the following.

1. $x^2 - 1$ [D] a. $(x+1)(x^2 - x + 1)$
2. $x^3 - 1$ [E] b. $(x^2 - x + 1)(x^2 + x + 1)$
3. $x^3 + 1$ [A] c. $(x+1)(x-1)(x^2 + 1)$
4. $x^4 - 1$ [C] d. $(x+1)(x-1)$
5. $x^4 - x^2 + 1$ [B] e. $(x-1)(x^2 + x + 1)$

IV. More than one option correct:

1. Factors of polynomial $12x^2 - 7x + 1$ are:
a) $(3x-1)(4x-1)$ b) $(4x-1)(3x-1)$ c) $(4x-1)(3x+1)$ d) None of these
Ans: A, B
2. Which of the following polynomial has a degree 3?
a) $2x^3 - 5x^2 + ax + b$ b) $6x^3 - 11x^2 + kx - 20$ c) $3x + 4$ d) None of these
Ans: A, B

3. Which of the following are false? **Ans: B, D**
 a) Highest power of the variable in a polynomial is the degree of the polynomial
 b) Degree of zero polynomial is always defined.
 c) A polynomial of degree one is called a linear polynomial.
 d) A polynomial of degree two is called a constant polynomial
4. Zeros of the polynomial $x(x + 7)$ are **Ans: A, C**
 a) -7 b) 7 c) 0 d) None of these
5. Which of the following are polynomials? **Ans: A, B, C**
 a) $x^3 + 4x^2 + 5x + 1$ b) $x^3 + 5x^2 + 2$ c) $x - 2 + 4$ d) None of these

V. Reason Type Questions:

Instruction: The questions given below, consists of two statements viz Statement-1 (assertion) and Statement-2 (reason). Use the following key to choose appropriate answer.

- (a) Both statements are correct and Statement-2 is the correct explanation of Statement-1.
 (b) Both statements are correct but Statement-2 is not correct explanation of Statement-1.
 (c) Statement-1 is true & Statement-2 is false.
 (d) Statement-1 is false & Statement-2 is true.
 (e) Both the statements are false.

- 1) **Statement 1 :** If $f(x) = x^3 + 6x^2 + 4x + 8$ is divided by $x + 2$ the remainder will be 0.
Statement 2 : $f(2) = 0$ **Ans: E**
- 2) **Statement 1 :** $x^3 + 1$ is a multiple of $x + 1$
Statement 2 : $x^3 - 1 = (x - 1)(x^2 + x + 1)$ **Ans: B**
- 3) **Statement 1 :** if $f(x) = x^4 - 1$ is divided by $x + 1$, the remainder will be 0.
Statement 2 : if $f(x) = x^4 - 1$ then $f(-1) = 0$ **Ans: A**
- 4) **Statement 1 :** if $x + y = 10$ and $xy = 10$ then $x^2 + y^2 = 100$
Statement 2 : $x^2 + y^2 = (x + y)(xy)$ **Ans: E**

VI. Very short answer questions.

1. Check whether -2 and 2 are the zeroes of the polynomial $x^4 - 16$.

Ans: $p(x) = x^4 - 16$

$$p(-2) = (-2)^4 - 16 = 16 - 16 = 0$$

$$p(2) = 2^4 - 16 = 16 - 16 = 0$$

Yes, -2 and 2 are the zeroes of the polynomial $x^4 - 16$

2. Find the quotient and remainder by dividing $x^3 - 1$ by $x - 1$.

Ans:

$$\begin{array}{r}
 x^2 + x + 1 \leftarrow \text{Quotient} \\
 x - 1 \overline{) x^3 - 1} \\
 \underline{x^3 - x^2} \\
 x^2 - 1 \\
 \underline{x^2 - x} \\
 x - 1 \\
 \underline{x - 1} \\
 0 \leftarrow \text{Remainder}
 \end{array}$$

3. Check whether -2 and 3 are the zeroes of the polynomial $x^2 - x - 6$.

Ans: $p(x) = x^2 - x - 6$

$$p(-2) = (-2)^2 - (-2) - 6 = 4 + 2 - 6 = 0$$

$$p(3) = 3^2 - 3 - 6 = 0$$

Yes, -2 and 3 are the zeros of the given polynomial.

4. Find the value of $f(3)$ if $f(x) = 3x^2 - 2x + 2$.

Ans: $f(x) = 3x^2 - 2x + 2$

$$f(3) = 3 \times 3^2 - 2 \times 3 + 2 = 23$$

5. Factorize: $x^2 - y^2 - 4xz + 4z^2$.

Ans: $x^2 - y^2 - 4xz + 4z^2 = (x - 2z)^2 - y^2 = (x - 2z + y)(x - 2z - y)$

6. Factorize: $49x^2 + 70xy + 25y^2$

Ans: $49x^2 + 70xy + 25y^2 = (7x)^2 + 2 \times 7x \times 5y + (5y)^2 = (7x + 5y)^2 = (7x + 5y)(7x + 5y)$

7. Find the zeros of the polynomial $p(x) = x^2 + 5x + 6$.

Ans: $p(x) = x^2 + 5x + 6$

$$= x^2 + 3x + 2x + 6$$

$$= x(x + 3) + 2(x + 3)$$

$$= (x + 2)(x + 3)$$

$$\therefore x + 2 = 0 \text{ or } x + 3 = 0$$

$$\Rightarrow x = -2 \text{ or } x = -3$$

So the zeros of $p(x)$ are -2 and -3

8. Find the zeros of the polynomial $p(x) = x^2 + 3x - 4$.

Ans: $p(x) = x^2 + 3x - 4 = (x + 4)(x - 1)$

So the zeros of $p(x)$ are -4 and 1

9. Find the zeros of the polynomial $p(x) = 6x^2 - 7x - 3$.

Ans: $p(x) = 6x^2 - 7x - 3 = (2x - 3)(3x + 1)$

So the zeros of $f(x)$ are $3/2$ and $1/3$

10. Find the quadratic polynomial for the zeroes $\frac{1}{2}$, $\frac{3}{2}$, given in each case.

Ans: $\alpha = \frac{1}{2}$, $\beta = \frac{3}{2}$

the equation is $x^2 - (\alpha + \beta)x + \alpha\beta$

$$x^2 - \left(\frac{1}{2} + \frac{3}{2}\right)x + \frac{1}{2} \times \frac{3}{2} = x^2 - 2x + \frac{3}{4} = 4x^2 - 8x + 3$$

VII. Short Answer Questions.

1. Evaluate the following products without multiplying directly.

i) 103×107

ii) 104×96

Ans: i) $103 \times 107 = (105 - 2)(105 + 2) = 105^2 - 2^2 = 11025 - 4 = 11021$

ii) $104 \times 96 = (100 + 4)(100 - 4) = 100^2 - 4^2 = 10000 - 16 = 9984$

2. Factorize: $6a^2b^2x - 48abx + 96x$

Ans: $6x(a^2b^2 - 8ab + 16) = 6x(ab + 4)^2$

3. Factorize: $4x^2 - 4\sqrt{3}x + 3$

Ans: $4x^2 - 4\sqrt{3}x + 3$

$$= 2((2x)^2 - 2 \times 2x \times \sqrt{3} + \sqrt{3}^2)$$

$$= 2(x - \sqrt{3})^2$$

4. Factorize: $12(a + 1)^2 - 25(a + 1)(b + 2) + 12(b + 2)^2$

Ans: $12(a + 1)^2 - 25(a + 1)(b + 2) + 12(b + 2)^2$

Let $a + 1 = x$ and $b + 2 = y$

$$= 12x^2 - 25xy + 12y^2 = 12x^2 - 16xy - 9xy + 12y^2 = 4x(3x - 4y) - 3y(3x - 4y)$$

$$= (3x - 4y)(4x - 3y) \text{ (Substituting } x \text{ value by } a + 1 \text{ and } y \text{ value by } b + 2)$$

$$= (3 \times (a + 1) - 4 \times (b + 2))(4 \times (a + 1) - 3 \times (b + 2))$$

$$= (3a + 3 - 4b - 8)(4a + 4 - 3b - 6) = (3a - 4b - 5)(4a - 3b - 2)$$

5. Find the zeroes of the quadratic polynomial and verify the relationship between the zeroes and the coefficients.

i) $x^2 - 2x - 8$

ii) $4x^2 - 4x + 1$

iii) $6x^2 - 3 - 7x$

Ans: i) $x^2 - 2x - 8 = x^2 - 4x + 2x - 8 = x(x - 4) + 2(x - 4) = (x - 4)(x + 2)$

so the zeros are 4 and -2

ii) $4x^2 - 4x + 1 = 4x^2 - 2x - 2x + 1 = 2x(2x - 1) - 1(2x - 1) = (2x - 1)(2x - 1)$

so the zeros are $\frac{1}{2}$ and $\frac{1}{2}$.

iii) $6x^2 - 3 - 7x = 6x^2 - 7x - 3 = 6x^2 + 2x - 9x - 3 = 2x(3x + 1) - 3(3x + 1) = (3x + 1)(2x - 3)$

so the zeros are $-\frac{1}{3}$ and $\frac{3}{2}$.

6. Find the quadratic polynomial for the zeroes α, β given in each case.

i) 2, -1

ii) $\sqrt{3}, -\sqrt{3}$

iii) $\frac{1}{4}, -1$

iv) $\frac{1}{2}, \frac{3}{2}$

Ans: i) $\alpha = 2, \beta = -1$

$$\begin{aligned}\text{Quadratic polynomial} &= x^2 - (\alpha + \beta)x + \alpha\beta = 0 \\ &\Rightarrow x^2 - (2 + (-1))x + 2 \times (-1) = 0 \\ &\Rightarrow x^2 - x - 2 = 0\end{aligned}$$

ii) $\alpha = \sqrt{3}, \beta = -\sqrt{3}$

$$\begin{aligned}\text{Quadratic polynomial} &= x^2 - (\alpha + \beta)x + \alpha\beta = 0 \\ &\Rightarrow x^2 - (\sqrt{3} + (-\sqrt{3}))x + \sqrt{3} \times (-\sqrt{3}) = 0 \\ &\Rightarrow x^2 - 3 = 0\end{aligned}$$

iii) $\alpha = \frac{1}{4}, \beta = -1$

$$\begin{aligned}\text{Quadratic polynomial} &= x^2 - (\alpha + \beta)x + \alpha\beta = 0 \\ &\Rightarrow x^2 - \left(\frac{1}{4} - 1\right)x + \frac{1}{4} \times (-1) = 0 \\ &\Rightarrow x^2 + \frac{3x}{4} - \frac{1}{4} = 0 \\ &\Rightarrow 4x^2 + 3x - 1 = 0\end{aligned}$$

iv) $\alpha = \frac{1}{2}, \beta = \frac{3}{2}$

$$\begin{aligned}\text{Quadratic polynomial} &= x^2 - (\alpha + \beta)x + \alpha\beta = 0 \\ &\Rightarrow x^2 - \left(\frac{1}{2} + \frac{3}{2}\right)x + \frac{1}{2} \times \frac{3}{2} = 0 \\ &\Rightarrow x^2 - 2x + \frac{3}{4} = 0 \\ &\Rightarrow 4x^2 - 8x + 3 = 0\end{aligned}$$

7. Divide the polynomial $x^3 - 3x^2 + 5x - 3$ by $x^2 - 2$ and find the quotient and remainder.

Ans:

$$\begin{array}{r} x - 3 \quad \leftarrow \text{Quotient} \\ x^2 - 2 \overline{) x^3 - 3x^2 + 5x - 3} \\ \underline{x^3 - 2x} \\ -3x^2 + 7x - 3 \\ \underline{-3x^2 + 6} \\ 7x - 9 \quad \leftarrow \text{Remainder} \end{array}$$

8. Find the value of $a^3 + b^3 + c^3 - 3abc$ given that $a + b + c = 10$ and $a^2 + b^2 + c^2 = 83$.

Ans: $a + b + c = 10$; $a^2 + b^2 + c^2 = 83$
 $\therefore 2(ab + bc + ca) = (a + b + c)^2 - (a^2 + b^2 + c^2) = 10^2 - 83 = 17$
 $\Rightarrow ab + bc + ca = 17/2 = 8.5$
 $\Rightarrow a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - (ab + bc + ca)) = 10 \times (83 - 8.5) = 745$

9. Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.

Ans: $7 + 3x = 0$
 $\Rightarrow x = \frac{-7}{3}$
 $f(x) = 3x^3 + 7x$
 $f\left(\frac{-7}{3}\right) = \left(\frac{-7}{3}\right)^3 + 7\left(\frac{-7}{3}\right) = -\frac{343}{27} - \frac{49}{3} = -\frac{784}{27}$
 So $7 + 3x$ is not a factor of $3x^3 + 7x$

10. If $A = x^3$, $B = 4x^2 + x - 1$, $C = x + 1$, then find $(A - B)(B - C)$.

Ans: $a = x^3$ $b = 4x^2 + x - 1$ $c = x + 1$
 $(a - b)(b - c) = (x^3 - 4x^2 - x + 1)(4x^2 + x - 1 - x - 1)$
 $= (x^3 - 4x^2 - x + 1)(4x^2 - 2)$
 $= 4x^5 - 16x^4 - 4x^3 + 4x^2 - 2x^3 - 8x^2 + 2x - 2$
 $= 4x^5 - 16x^4 - 6x^3 - 4x^2 + 2x - 2$

VIII. Long Answer Questions.

1. Find the zeroes of the polynomial $2(x + 3)^2 + 9(x + 3)$.

Ans: $2(x + 3)^2 + 9(x + 3) = (x + 3)(2x + 6 + 9) = (x + 3)(2x + 15)$
 $x + 3 = 0$ or $2x + 15 = 0$
 $x = -3$ or $x = 15/2 = 7.5$
 So the zeros of the polynomial are -3 and -7.5

2. Show that $3x - 2$ is a factor of $3x^3 + x^2 - 20x + 12$.

Ans: if $3x - 2 = 0 \Rightarrow$ then $x = \frac{2}{3}$
 $f(x) = 3x^3 + x^2 - 20x + 12$
 $f\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^2 - 20\left(\frac{2}{3}\right) + 12$
 $\Rightarrow \left(\frac{8}{9}\right) + \left(\frac{4}{9}\right) - \left(\frac{40}{3}\right) + 12 = 0$

So $3x - 2$ is a factor of $3x^3 + x^2 - 20x + 12$

3. If $2x^3 + ax^2 + 11x + 2$ is a multiple of $2x - 1$ then find then the value of a .

Ans: $f(x) = 2x^3 + ax^2 + 11x + 2$

$$g(x) = 2x - 1 = 0 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$$

As $f(x)$ is a multiple of $g(x)$

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 + 11\left(\frac{1}{2}\right) + 2 = 0$$

$$\Rightarrow \frac{1}{4} + \frac{a}{4} + \frac{11}{2} + 2 = 0$$

$$\Rightarrow \frac{1 + a + 22 + 8}{4} = 0$$

$$\Rightarrow 31 + a = 0$$

$$\Rightarrow a = -31$$

4. If 2 and $\frac{1}{2}$ are the zeros of the polynomial $px^2 + 5x + r$ then prove that $p = r$.

Ans: $f(x) = px^2 + 5x + r$ the roots are 2 and $\frac{1}{2}$

The product of the roots $\frac{r}{p} = 1 \Rightarrow p = r$

5. If the zeros of the polynomial $x^3 - 3x^2 + x + 1$ are $a - b$, a and $a + b$ then find a and b .

Ans: $f(x) = x^3 - 3x^2 + x + 1$ the roots are $a - b$, a , and $a + b$

sum of the roots $= a - b + a + a + b = -3/1$

$$\Rightarrow 3a = -3 \Rightarrow a = -3/3 = -1$$

Again product of the roots $= (a - b)(a + b)a = 1/1$

$$\Rightarrow a^3 - ab^2 = 1 \text{ (Substituting a value by -1)}$$

$$\Rightarrow (-1)^3 - (-1) \times b^2 = 1 \Rightarrow -1 + b^2 = 1$$

$$\Rightarrow b^2 = 2 \Rightarrow b = \pm\sqrt{2}$$

6. If $x = 4/3$ is a root of the polynomial $6x^3 - 11x^2 + kx - 20$ then find the value of k .

Ans: if $x = 4/3$ is a root of $6x^3 - 11x^2 + kx - 20$

Let $f(x) = 6x^3 - 11x^2 + kx - 20$

$$f\left(\frac{4}{3}\right) = 6\left(\frac{4}{3}\right)^3 - 11\left(\frac{4}{3}\right)^2 + kx - 20 = 0$$

$$\Rightarrow \frac{128}{9} - \frac{176}{9} + \frac{4k}{3} - 20 = 0$$

$$\Rightarrow \frac{128 - 176 + 12k - 180}{9} = 0$$

$$\Rightarrow 12k - 228 = 0 \Rightarrow 12k = 228 \Rightarrow k = \frac{228}{12} = 19$$

7. Factorize: $x^4 + x^3 - 7x^2 - x + 6$

Ans: $x^4 + x^3 - 7x^2 - x + 6$

Here we have $p(x) = x^4 + x^3 - 7x^2 - x + 6$ (say)

Here we have constant term 6

and coefficient of x^4 is 1,

So, the factors of constant term i.e. 6 are $\pm 1, \pm 2, \pm 3, \pm 6$

Case 1: Put $x = 1$ and check if that satisfied our equation $x^4 + x^3 - 7x^2 - x + 6$

$$P(1) = (1)^4 + (1)^3 - 7(1)^2 - (1) + 6$$

$$= 1 + 1 - 7 - 1 + 6$$

$$= 0$$

So $(x - 1)$ is a factor of the given equation $p(x)$.

[Factor theorem, if $f(a) = 0$, then $(x - a)$ is a factor of polynomial $f(x)$]

Case 2: Put $x = -1$ and check if that satisfied our equation, $x^4 + x^3 - 7x^2 - x + 6$

$$P(-1) = (-1)^4 + (-1)^3 - 7(-1)^2 - (-1) + 6$$

$$= 1 - 1 - 7 + 1 + 6$$

$$= 0$$

So $(x + 1)$ is a factor of the given equation $p(x)$.

Case 3: Put $x = 2$ and check if that satisfied our equation, $x^4 + x^3 - 7x^2 - x + 6$

$$P(2) = (2)^4 + (2)^3 - 7(2)^2 - (2) + 6$$

$$= 16 + 8 - 28 - 2 + 6$$

$$= 0$$

So $(x - 2)$ is a factor of the given equation $p(x)$.

Case 4: Put $x = -2$ and check if that satisfied our equation, $x^4 + x^3 - 7x^2 - x + 6$

$$P(-2) = (-2)^4 + (-2)^3 - 7(-2)^2 - (-2) + 6$$

$$= 16 - 8 - 28 + 2 + 6$$

$$= -12$$

$$\neq 0$$

So $(x + 2)$ is not a factor of the given equation $p(x)$.

Case 5: Put $x = -3$ and check if that satisfied our equation, $x^4 + x^3 - 7x^2 - x + 6$

$$P(-3) = (-3)^4 + (-3)^3 - 7(-3)^2 - (-3) + 6$$

$$= 81 - 27 - 63 + 3 + 6$$

$$= 0$$

So $(x + 3)$ is a factor of the given equation $p(x)$.

So we have, the factors of $x^4 + x^3 - 7x^2 - x + 6$ are $(x - 1)(x + 1)(x - 2)(x + 3)$

8. In a division algorithm, the divisor is $x^3 + 3$, quotient is $x - 1$ and the remainder is 6. What is the dividend?

Ans: dividend = divisor $\times$ quotient + remainder

$$=(x^3 + 3)(x - 1) + 6 = x^4 - x^3 + 3x - 3 + 6 = x^4 - x^3 + 3x + 3$$

GOLD PACKAGE



I. Choose the correct answer.

- 1) If we divide $3y^4 - y^3 + 12y^2 + 2$ by $3y^2 - 1$ the remainder is

a) $\frac{y}{3} + \frac{19}{3}$

b) $-\frac{y}{3} + \frac{19}{3}$

c) $-\frac{y}{3} - \frac{19}{3}$

d) None of these

Ans: (B)

$$\begin{array}{r}
 3y^2 - 1 \overline{) 3y^4 - y^3 + 12y^2 + 2} \\
 \underline{3y^4 - y^2} \\
 -y^3 + 13y^2 + 2 \\
 \underline{-y^3 + (1/3)y} \\
 13y^2 - (1/3)y + 2 \\
 \underline{13y^2 - (13/3)} \\
 -(1/3)y + (19/3)
 \end{array}$$

← Quotient

← Remainder

2) $\left(a^{\frac{1}{8}} + b^{\frac{1}{8}}\right)\left(a^{\frac{1}{8}} - b^{\frac{1}{8}}\right)\left(a^{\frac{1}{4}} + b^{\frac{1}{4}}\right)\left(a^{\frac{1}{2}} + b^{\frac{1}{2}}\right) =$

a) $a + b^{-1}$

b) $a - b^{-1}$

c) $a - b$

d) None of these

Ans: (C)

$$\begin{aligned}
 &\left(a^{\frac{1}{8}} + b^{\frac{1}{8}}\right)\left(a^{\frac{1}{8}} - b^{\frac{1}{8}}\right)\left(a^{\frac{1}{4}} + b^{\frac{1}{4}}\right)\left(a^{\frac{1}{2}} + b^{\frac{1}{2}}\right) \\
 &= \left(a^{\frac{1}{8}} - b^{\frac{1}{8}}\right)^2 \left(a^{\frac{1}{4}} + b^{\frac{1}{4}}\right)\left(a^{\frac{1}{2}} + b^{\frac{1}{2}}\right) \\
 &= \left(a^{\frac{1}{4}} - b^{\frac{1}{4}}\right)\left(a^{\frac{1}{4}} + b^{\frac{1}{4}}\right)\left(a^{\frac{1}{2}} + b^{\frac{1}{2}}\right) \\
 &= \left(a^{\frac{1}{4}} - b^{\frac{1}{4}}\right)^2 \left(a^{\frac{1}{2}} + b^{\frac{1}{2}}\right) \\
 &= \left(a^{\frac{1}{2}} - b^{\frac{1}{2}}\right)\left(a^{\frac{1}{2}} + b^{\frac{1}{2}}\right) = \left(a^{\frac{1}{2}} - b^{\frac{1}{2}}\right)^2 = a - b
 \end{aligned}$$

3) $\left(x - \frac{1}{x}\right)^2 = x^2 + x + \frac{1}{x^2}$ than the value of x

a) -2

b) 2

c) 2x

d) None of these

Ans: (A)

$$\left(x - \frac{1}{x}\right)^2 = x^2 + x + \frac{1}{x^2}$$

$$\Rightarrow x^2 + \frac{1}{x^2} - 2 = x^2 + x + \frac{1}{x^2}$$

$$\Rightarrow x^2 + \frac{1}{x^2} - 2 - x^2 - x - \frac{1}{x^2} = 0$$

$$\Rightarrow -x = 2 \Rightarrow x = -2$$

4) Factors of $x^2 - 7x + 12$ are:

a) $(x-3)(x+4)$

b) $(x-3)(x-4)$

c) $(x+3)(x+4)$

d) None of these

Ans: (B)

$$x^2 - 7x + 12$$

$$= x^2 - 3x - 4x + 12$$

$$= x(x-3) - 4(x-3) = (x-4)(x-3)$$

5) If $x = 2$ and $y = -1$ then the value of $x^2 + 4xy + 4y^2$ is:

a) 0

b) 1

c) -1

d) None of these

Ans: (A)

$$x = 2 \text{ and } y = -1$$

$$x^2 + 4xy + 4y^2$$

$$= 2^2 + 4 \times 2 \times -1 + 4 \times (-1)^2$$

$$= 4 - 8 + 4 = 0$$

6) Factors of $x^3 - 3x^2 - 10x + 24$ are:

a) $(x-2)(x+3)(x-4)$

b) $(x+2)(x+3)(x+4)$

c) $(x+2)(x-3)(x-4)$

d) None of these

Ans: (A)

$$x^3 - 3x^2 - 10x + 24$$

$$= x^3 + x^2 - 6x - 4x^2 - 4x + 24$$

$$= x(x^2 + x - 6) - 4(x^2 + x - 6)$$

$$= (x-4)(x^2 + x - 6)$$

$$= (x-4)(x^2 - 2x + 3x - 6)$$

$$= (x-4)(x(x-2) + 3(x-2))$$

$$= (x-4)(x+3)(x-2)$$

7) The value of a and b so that the polynomial $x^3 - ax^2 - 13x + b$ is divisible by $x + 1$ and $x - 3$ are:

a) $a = 15, b = 3$

b) $a = 0, b = -12$

c) $a = -3, b = -15$

d) None of these

Ans: (C)

Sol: let $f(x) = x^3 - ax^2 - 13x + b$

As the polynomial $x^3 - ax^2 - 13x + b$ is divisible by $x + 1$ and $x - 3$

$$f(-1) = (-1)^3 - a(-1)^2 - 13(-1) + b = 0 \text{ -----(1)}$$

$$F(3) = 3^3 - a \times 3^2 - 13 \times 3 + b = 0 \text{ -----(2)}$$

$$f(-1) = f(3)$$

$$\Rightarrow (-1)^3 - a(-1)^2 - 13(-1) + b = 3^3 - a \times 3^2 - 13 \times 3 + b$$

$$\Rightarrow -1 - a + 13 + b = 27 - 9a - 39 + b$$

$$\Rightarrow -a + b + 9a - b = -12 - 12$$

$$\Rightarrow 8a = -24$$

$$\Rightarrow a = -24/8 = -3$$

Now substituting a value by -3 in equation (1)

$$(-1)^3 - a(-1)^2 - 13(-1) + b = 0$$

$$\Rightarrow -1 - (-3) + 13 + b = 0$$

$$\Rightarrow 15 + b = 0$$

$$\Rightarrow b = -15$$

8) Factors of $a^2 - b + ab - a$ are:

a) $(a - b)(a + 1)$

b) $(a + b)(a - 1)$

c) $(a - b)(a - 1)$

d) None of these

Ans: (B)

Sol: $a^2 - b + ab - a = a^2 + ab - a - b = a(a + b) - 1(a + b) = (a - 1)(a + b)$

9) If one factor of $a(x + y + z) + bx + by + bz$ is $(x + y + z)$ then the other factor is:

a) $ax + ay + az$

b) $bx + by + bz$

c) $bx + by - bz$

d) none of these

Ans: (D) None of these

Sol: $a(x + y + z) + bx + by + bz$

$$= a(x + y + z) + b(x + y + z)$$

$$= (a + b)(x + y + z)$$

10) If $2x^2 + xy - 3y^2 + x + ay - 10 = (2x + 3y + b)(x - y - 2)$ then the value of a and b are:

a) -11 and 5

b) 1 and -5

c) -1 and -5

d) none of these

Ans: (D) None of these

Sol: $2x^2 + xy - 3y^2 + x + ay - 10 = (2x + 3y + b)(x - y - 2)$

$$\Rightarrow 2x^2 + xy - 3y^2 + x + ay - 10 = 2x^2 - 2xy - 4x + 3xy - 3y^2 - 6y + bx - by - 2b$$

$$\Rightarrow 2x^2 + xy - 3y^2 + x + ay - 10 = 2x^2 + xy - 3y^2 + (b - 4)x - (b + 6)y - 2b$$

Comparing the coefficients of x, y and constants will get:

$$b - 4 = 1 \dots\dots\dots(i)$$

$$-(b + 6) = a \dots\dots\dots(ii)$$

$$-2b = -10 \dots\dots\dots(iii)$$

From eq(1), we get

$$\Rightarrow b = 5$$

Now put b in eq(2), we get

$$\Rightarrow -(b + 6) = a$$

$$\Rightarrow a = -11$$

From eq(3), we get $\Rightarrow -2b = -10$

$$\Rightarrow b = 5$$

Hence values of a, b are $-11, 5$

II. Choose the correct answer. (More than one answer)

1) If the zeros of the quadratic equation are 11 and 2, what is expression?

a) $x^2 - 13x + 22$

b) $x^2 - 11x + 22$

c) $2x^2 - 26x + 44$

d) $x^2 + 13x - 22$

Sol: $\alpha = 11, \beta = 2$

$$\text{Quadratic equation} = x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\Rightarrow x^2 - (11 + 2)x + 11 \times 2 = 0$$

$$\Rightarrow x^2 - 13x + 22 = 0 \text{ (Multiplying both side by 2)}$$

$$\Rightarrow 2x^2 - 26x + 44 = 0 \text{ Ans: (A), (C)}$$

2) If $f(x) = 2x^3 - x^2 - 2x + 1$ then the zeros of the polynomial are

a) $\frac{1}{2}$

b) 1

c) $-\frac{1}{2}$

d) None of these

Sol: $2x^3 - x^2 - 2x + 1 = 0$

$$\Rightarrow x^2(2x - 1) - 1(2x - 1) = 0$$

$$\Rightarrow (x^2 - 1)(2x - 1) = 0 \Rightarrow x^2 - 1 = 0 \text{ or } 2x - 1 = 0$$

$$\Rightarrow x = 1 \text{ or } x = \frac{1}{2} \text{ Ans: (A), (B)}$$

3) If the zeros of the quadratic polynomial $ax^2 + bx + c, a \neq 0$ are equal, then

a) c and a have opposite signs

b) c and b have opposite signs

c) c and a have same signs

d) c and b have same signs

Ans: (B), (C)

4) What are the zeros of the polynomial $6x^2 - 7x - 3$?

a) $\frac{3}{2}, -\frac{1}{3}$

b) $\frac{3}{2}, \frac{1}{3}$

c) $-\frac{3}{2}, \frac{1}{3}$

d) $1.5, -\frac{1}{3}$

Sol: $6x^2 - 7x - 3 = 0 \Rightarrow 6x^2 - 9x + 2x - 3 = 0 \Rightarrow 3x(2x - 3) + 1(2x - 3) = 0$

$$(3x + 1)(2x - 3) = 0 \Rightarrow 3x + 1 = 0 \text{ or } 2x - 3 = 0 \Rightarrow x = -\frac{1}{3} \text{ or } x = \frac{3}{2} \text{ Ans: (A), (D)}$$

5) The zeros of the polynomial $x^4 - 3x^3 + 6x - 4$

a) $\sqrt{2}$

b) $-\sqrt{2}$

c) $\sqrt{3}$

d) None of these

Sol: $x^4 - 3x^3 + 6x - 4 = 0$

$$\Rightarrow x^4 - x^3 - 2x^2 + 2x - 2x^3 + 2x^2 + 4x - 4 = 0$$

$$\Rightarrow x(x^3 - x^2 - 2x + 2) - 2(x^3 - x^2 - 2x + 2) = 0$$

$$\Rightarrow (x - 2)(x^3 - x^2 - 2x + 2) = 0$$

$$\Rightarrow (x - 2)(x^2(x - 1) - 2(x - 1)) = 0$$

$$\Rightarrow (x - 2)(x^2 - 2)(x - 1) = 0$$

$$\therefore x - 2 = 0 \quad \text{or} \quad x^2 - 2 = 0 \quad \text{or} \quad x - 1 = 0$$

$$\Rightarrow x = 2 \quad \text{or} \quad x = \pm\sqrt{2} \quad \text{or} \quad x = 1 \quad \text{Ans: (A), (B)}$$

III. Answer the following questions.

1) If $p(x) = x^2 - 4x + 3$, evaluate: $p(2) - p(-1) + p(\frac{1}{2})$

Ans: $p(x) = x^2 - 4x + 3$

$$p(2) = 2^2 - 4 \times 2 + 3 = -1$$

$$p(-1) = (-1)^2 - 4 \times -1 + 3 = 8$$

$$p(\frac{1}{2}) = (\frac{1}{2})^2 - 4 \times \frac{1}{2} + 3 = \frac{1}{4} + 1 = \frac{5}{4}$$

$$\therefore p(2) - p(-1) + p(\frac{1}{2}) = -1 - 8 + \frac{5}{4} = -9 + \frac{5}{4} = -\frac{31}{4}$$

2) Find the zeros of the polynomial $x^4 - 1$

Ans: $x^4 - 1 = 0$

$$\Rightarrow (x^2)^2 - (1^2)^2 = 0$$

$$\Rightarrow (x^2 + 1^2)(x^2 - 1^2) = 0$$

$$\Rightarrow (x^2 + 1)(x - 1)(x + 1) = 0$$

$$\therefore x^2 + 1 = 0 \quad \text{or} \quad x - 1 = 0 \quad \text{or} \quad x + 1 = 0$$

$$\Rightarrow x = \sqrt{-1} \quad \text{or} \quad x = 1 \quad \text{or} \quad x = -1$$

3) Without actual division, prove that $2x^4 - 6x^3 + 3x^2 + 3x - 2$ is exactly divisible by $x^2 - 3x + 2$.

Ans: $p(x) = 2x^4 - 6x^3 + 3x^2 + 3x - 2$

$$q(x) = x^2 - 3x + 2 = (x - 2)(x - 1)$$

$$p(2) = 2 \times 2^4 - 6 \times 2^3 + 3 \times 2^2 + 3 \times 2 - 2 = 0$$

$$p(1) = 2 \times 1^4 - 6 \times 1^3 + 3 \times 1^2 + 3 \times 1 - 2 = 0$$

so $p(x)$ is divisible by $q(x)$

4) Find the value of $\left(x^4 - \frac{1}{x^4}\right)$ if $\left(x - \frac{1}{x}\right) = 4$

Ans: $\left(x - \frac{1}{x}\right) = 4$ $\Rightarrow \left(x - \frac{1}{x}\right)^2 = 16$ $\Rightarrow x^2 + \frac{1}{x^2} - 2 = 16$ $\Rightarrow x^2 + \frac{1}{x^2} = 16 + 2 = 18$ $\Rightarrow \left(x + \frac{1}{x}\right)^2 - 2 = 18$ $\Rightarrow x + \frac{1}{x} = \sqrt{20}$	$\left(x^4 - \frac{1}{x^4}\right)$ $= \left(x^2 + \frac{1}{x^2}\right)\left(x^2 - \frac{1}{x^2}\right)$ $= \left(x^2 + \frac{1}{x^2}\right)\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$ $= 18 \times \sqrt{20} \times 4$ $= 144\sqrt{5}$
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5) Factorize: $\frac{a^2}{4b^2} - \frac{1}{3} + \frac{b^2}{9a^2}$

Ans: $\frac{a^2}{4b^2} - \frac{1}{3} + \frac{b^2}{9a^2} = \left(\frac{a}{2b}\right)^2 - 2 \times \frac{a}{2b} \times \frac{b}{3a} + \left(\frac{b}{3a}\right)^2$

$= \left(\frac{a}{2b} - \frac{b}{3a}\right)^2 = \left(\frac{a}{2b} - \frac{b}{3a}\right)\left(\frac{a}{2b} - \frac{b}{3a}\right)$

6) Factorize: $a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)$

Ans: $a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)$

$= ab^2 - ac^2 + bc^2 - ba^2 + ca^2 - cb^2$

$= ab^2 - a^2b + ca^2 - c^2a + bc^2 - b^2c$

$= -ab(a - b) - c^2(a - b) + c(a + b)(a - b)$

$= (a - b)(-ab - c^2 + ac + bc)$

$= (a - b)(-ab + ac - c^2 + bc)$

$= (a - b)(-a(b - c) + c(b - c)) = (a - b)(b - c)(c - a)$

7) Factorize: $a^3(b - c) + b^3(c - a) + c^3(a - b)$

Ans: $a^3(b - c) + b^3(c - a) + c^3(a - b) = a^3b - a^3c + b^3c - b^3a + c^3a - c^3b$

$= a^3b - ab^3 - a^3c + b^3c + c^3a - c^3b = ab(a^2 - b^2) - c(a^3 - b^3) + c^3(a - b)$

$= (a - b)\{a^2(b - c) + ab(b - c) - c(b + c)(b - c)\} = (a - b)(b - c)(a^2 + ab - bc - c^2)$

$= (a - b)(b - c)(a - c)(a + b + c)$

8) Factorize: $ab(x^2 - y^2) + xy(b^2 - a^2)$

Ans: $ab(x^2 - y^2) + xy(b^2 - a^2)$
 $= abx^2 - aby^2 + xyb^2 - xya^2$
 $= ax(bx - ay) + yb(bx - ay)$
 $= (bx - ay)(ax + yb)$

9) If $\left(x + \frac{1}{x}\right) = 6$ then find the value of $\left(x^3 + \frac{1}{x^3}\right)$

Ans: $\left(x^3 + \frac{1}{x^3}\right) = \left(x + \frac{1}{x}\right) \left\{ \left(x + \frac{1}{x}\right)^2 - 3 \right\} = 6(6^2 - 3) = 198$

10) Find the value of m if $16x^4 - 24x^3 + 41x^2 - mx + 16$ is a perfect square.

Ans: let $16x^4 - 24x^3 + 41x^2 - mx + 16 = (ax^2 + bx + c)^2$
 $\Rightarrow 16x^4 - 24x^3 + 41x^2 - mx + 16 = a^2x^4 + b^2x^2 + c^2 + 2abx^3 + 2bcx + 2acx^2$
 $[(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$
 $\Rightarrow 16x^4 - 24x^3 + 41x^2 - mx + 16 = a^2x^4 + 2abx^3 + (2ca + b^2)x^2 + 2bcx + c^2$
Comparing the coefficient of x on both sides, we get
 $c^2 = 16$ so $a = 4$
 $c^2 = 16$ so $c = 4$
 $2ab = -24$ so $b = -3$
 $2bc = -m$ so $m = 24$

EXERCISE - 2**LINEAR EQUATION WITH TWO VARIABLES****I. Multi choice Questions:**

1. If (2, 0) is a solution of the linear equation $2x + 3y = k$, then the value of k is
 a) 4 b) 6 c) 5 d) None of these

Sol: (2, 0) is a solution of the linear equation $2x + 3y = k$

$$\therefore x = 2 \text{ and } y = 0$$

$$2x + 3y = k \Rightarrow 2 \times 2 + 3 \times 0 = k \Rightarrow k = 4$$

2. Which of the following system of equations has infinitely many solutions:
 a) $5x - 4y = 20$, $7.5x - 6y = 30$ b) $2x - 3y = 5$, $3x - 4.5y = 7.5$
 c) $x + 5y - 5 = 0$, $3x + 7.5y - 4.5 = 0$ d) None of these

Sol: To have infinitely many solutions, it must satisfy the condition

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Option (a): $5x - 4y = 20$, $7.5x - 6y = 30$

$$\frac{a_1}{a_2} = \frac{5}{7.5} = \frac{50}{75} = \frac{2}{3} \quad \frac{b_1}{b_2} = \frac{-4}{-6} = \frac{2}{3} \quad \frac{c_1}{c_2} = \frac{20}{30} = \frac{2}{3}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad (\text{Infinitely many solutions})$$

Option (b): $2x - 3y = 5$, $3x - 4.5y = 7.5$

$$\frac{a_1}{a_2} = \frac{2}{3} \quad \frac{b_1}{b_2} = \frac{-3}{-4.5} = \frac{30}{45} = \frac{2}{3} \quad \frac{c_1}{c_2} = \frac{5}{7.5} = \frac{50}{75} = \frac{2}{3}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad (\text{Infinitely many solutions})$$

Option (c): $x + 5y - 5 = 0$, $3x + 7.5y - 4.5 = 0$

$$\frac{a_1}{a_2} = \frac{1}{3} \quad \frac{b_1}{b_2} = \frac{5}{7.5} = \frac{50}{75} = \frac{2}{3} \quad \frac{c_1}{c_2} = \frac{-5}{-4.5} = \frac{50}{45} = \frac{10}{9}$$

$$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \quad (\text{A unique solution})$$

Ans: (A), (B)

3. Which of the following is not a possible solution for $2x - 5y = 4$?
 a) (2, 0) b) (7, 2) c) (15, 6) d) None of these

Sol: **Option (A) (2, 0)**

$$2x - 5y = 4 \quad \Rightarrow 2 \times 2 - 5 \times 0 = 4 \quad \Rightarrow 4 = 4 \quad (\text{Possible solution})$$

Option (B) (7, 2)

$$2x - 5y = 4 \quad \Rightarrow 2 \times 7 - 5 \times 2 = 4 \quad \Rightarrow 4 = 4 \text{ (Possible solution)}$$

Option (A) (15, 6)

$$2x - 5y = 4 \quad \Rightarrow 2 \times 15 - 5 \times 6 = 4 \quad \Rightarrow 0 \neq 4 \text{ (Not Possible solution)}$$

Ans: (C) (15, 6)

4) For what value of k the system of equations $kx - y = 2$, $6x - 2y = 3$ has infinitely many solutions:

- a) $k = 3$ b) $k = 4$ c) $k = 6$ d) None of these

Sol: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (infinitely many solutions)

$$\Rightarrow \frac{k}{6} = \frac{-1}{-2} \neq \frac{2}{3} \quad \text{(So this is wrong)}$$

Ans: (D) None of these

5) Given that $2y = -6$ and $x - 3y = 13$, the value of x is:

- a) $13/9$ b) 4 c) 19 d) None of these

Sol: $2y = -6 \Rightarrow y = -6/2 = -3$

Now using the value of y in $x - 3y = 13$

$$x - 3 \times -3 = 13$$

$$\Rightarrow x = 13 - 9 = 4$$

Ans: (B) 4

6) For what value of k will the equations $2x + ky = 1$ and $5x - 7y = 5$ represent two parallel lines:

- a) $-\frac{14}{5}$ b) $\frac{14}{5}$ c) $-\frac{17}{5}$ d) None of these

Sol: $2x + ky = 1$

$$5x - 7y = 5$$

If the above lines are to be parallel, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\frac{2}{5} = \frac{k}{-7} \neq \frac{1}{5}$$

$$\Rightarrow 5k = -14 \Rightarrow k = \frac{-14}{5}$$

Ans: (A) $-\frac{14}{5}$

7) Given that $4x - 3y = 6$ and $x - 3y = -3$, the value of y is:

- a) -3 b) -2 c) 2 d) None of these

Sol: $4x - 3y = 6$ (1)

$$x - 3y = -3 \quad \text{..... (2)}$$

Equation (1) - (2)

$$4x - 3y = 6$$

$$\begin{array}{r} x - 3y = -3 \\ - \quad + \quad + \end{array}$$

$$\boxed{3x = 9} \Rightarrow x = 9/3 = 3$$

Now substituting x value by 3 in equation (1)

$$4x - 3y = 6$$

$$\Rightarrow 4 \times 3 - 3y = 6 \Rightarrow -3y = 6 - 12 = -6 \Rightarrow y = -6/-3 = 2$$

Ans: (C) 2

8. The lines $3x - 2y = 1$, $4x + 3y = 7$ and $x = 1$ intersect in:

- a) one point b) two points c) infinite number of points d) None of these

Sol: Use $x = 1$

$$3x - 2y = 1 \Rightarrow 3 \times 1 - 2y = 1 \Rightarrow -2y = 1 - 3 = -2 \Rightarrow y = -2/-2 = 1$$

$$4x + 3y = 7 \Rightarrow 4 \times 1 + 3y = 7 \Rightarrow 3y = 7 - 4 = 3 \Rightarrow y = 3/3 = 1$$

$\therefore$ It intersect in one point

Ans: (A) One point

9. Hari bought 30 stamps consisting of 20 paise and 30 paise stamps. Given that the stamps cost Rs. 7.80 altogether, how many 20 paise stamps did he buy?

- a) 12 b) 14 c) 10 d) None of these

Sol: Let number of 20 paise stamps = x

$\therefore$ Number of 30 paise stamps = $30 - x$

Total amount of 20 paise stamps = $x \times 20 = 20x$ paise

Total amount of 30 paise stamps = $(30 - x) 30 = 900 - 30x$ paise

Given that the stamps cost Rs. 7.80 (780 paise) altogether.

$$\therefore 20x + 900 - 30x = 780$$

$$\Rightarrow -10x = 780 - 900 = -120$$

$$\Rightarrow x = -120/-10 = 12$$

Number of 20 paise stamps = 12

Ans: (A) 12

10. Linear equation $2x + 3 = 7$ is:

- a) parallel to x-axis b) parallel to y-axis c) passes through the origin d) None of these

Ans: (B) Parallel to y-axis

II. Say True or False.

1. A linear equation $ax+by+c=0$ where a, b, c are real numbers ($a, b \neq 0$), has an unique solution.
2. A linear equation $ax + by + c = 0$ where a, b, c are real numbers and $a = 0$, the solution line will be parallel to the x axis.
3. We cannot draw a graph of the solution to a linear equation with two variables.
4. We cannot get a solution graphically to a pair of linear equations with two variables.
5. The line of the equation $x = a$ is perpendicular to the x -axis.
6. The line of the equation $y = a$ is parallel to the x -axis.
7. The points $(3,0)$, $(5,0)$ and $(0,6)$ lie on a straight line.
8. In a system of two linear equations, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ then the system has infinite number of solutions.
9. In a system of two linear equations, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ then the system has a unique solution.
10. The equation $x = 0$ represents the x axis.

Ans: 1) true 2) true 3) false 4) false 5) true
6) true 7) false 8) false 9) false 10) false

III. Match the following.

- | | | |
|------------------------------------|-------|--------------------------------|
| 1. $x = 4$ and $y = 5$ | [B] | a) parallel to each other |
| 2. $2x + 3y = 4$ and $4x + 6y = 5$ | [A] | b) perpendicular to each other |
| 3. $x + 2y = 1$ and $3x + 6y = 3$ | [D] | c) unique solution |
| 4. $y = mx$ | [E] | d) infinite solutions |
| 5. $x + 2y = 1$ and $2x + 3y = 5$ | [C] | e) passes through origin |

IV. More than one answer type questions.

1. Given that $x + 2y = 4$ and $2x + y = 5$ then
 - a) The lines are parallel
 - b) The lines are intersecting
 - c) The lines are perpendicular
 - d) $(2, 1)$ is the solution

Sol: Here, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (So, lines are intersecting)

$$x + 2y = 4 \Rightarrow x = 4 - 2y$$

Substituting x value by $4 - 2y$ in $2x + y = 5$

$$2 \times (4 - 2y) + y = 5 \Rightarrow 8 - 4y + y = 5 \Rightarrow -3y = 5 - 8 = -3 \Rightarrow y = -3/-3 = 1$$

Now Substituting y value by 1 in $x = 4 - 2y$

$$x = 4 - 2 \times 1 = 4 - 2 = 2$$

Ans: B, D

V. Very short answer type questions.

1. How many solutions does the pair of equations $y = 0$ and $y = -5$ have?

Ans: No Solution

2. If $ax + by = a^2 - b^2$ and $bx + ay = 0$, find the value of $(x + y)$.

Sol: $ax + by = a^2 - b^2$ (1)

$bx + ay = 0$ (2)

Adding (1) and (2)

$$ax + by + bx + ay = a^2 - b^2$$

$$\Rightarrow ax + bx + ay + by = a^2 - b^2$$

$$\Rightarrow x(a + b) + y(a + b) = a^2 - b^2$$

$$\Rightarrow (x + y)(a + b) = a^2 - b^2$$

$$\Rightarrow (x + y) = a - b$$

Ans: $a - b$

3. For what value of k , the pair of equations $4x - 3y = 9$, $2x + ky = 11$ has no solution?

Sol: No solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\Rightarrow \frac{4}{2} = \frac{-3}{k} \neq \frac{9}{11}$$

$$\Rightarrow 4k = -6$$

$$\Rightarrow k = \frac{-6}{4} = \frac{-3}{2}$$

Ans: $\frac{-3}{2}$

4. Find whether the following pair of linear equations is consistent or inconsistent:

$$3x + 2y = 8; 6x - 4y = 9$$

Sol: If the pair of linear equations has a unique solution, it is called consistent.

A unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\text{Here, } \frac{3}{6} \neq \frac{2}{-4} \Rightarrow \frac{1}{2} \neq \frac{1}{-2}$$

Ans: Consistent

5. Find the value of k for which the system of equations have infinitely many solutions:

$$x - ky = 2; 3x + 6y = 6$$

Sol: Infinitely many solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{1}{3} = \frac{-k}{6} = \frac{2}{6} \Rightarrow k = -2$

Ans: -2

6. What is the value of x which satisfies the equation $2x - (4 - x) = 5 - x$?

Sol: $2x - (4 - x) = 5 - x$
 $\Rightarrow 2x - 4 + x + x = 5$
 $\Rightarrow 4x = 5 + 4 = 9$
 $\Rightarrow x = 9/4$

7. For what positive values of ' p ' the following pair of linear equations have infinitely many solutions?
 $px + 3y - (p - 3) = 0$; $12x + py - p = 0$

Sol: Infinitely many solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{p}{12} = \frac{3}{p} = \frac{-(p-3)}{-p} \Rightarrow p = 6$

8. What is the value of x if $\frac{1}{7} + \frac{x}{7} = 3$

Sol: $\frac{1}{7} + \frac{x}{7} = 3$
 $\Rightarrow \frac{1+x}{7} = 3 \Rightarrow x + 1 = 3 \times 7 = 21 \Rightarrow x = 21 - 1 = 20$

9. If $-4(-t) + 3t = 35$ then find the value of t .

Sol: $-4(-t) + 3t = 35 \Rightarrow 4t + 3t = 35 \Rightarrow 7t = 35 \Rightarrow t = 35/7 = 5$

10. If $\frac{y}{3} = y - 4$ then what is the value of y ?

Sol: $\frac{y}{3} = y - 4$
 $\Rightarrow y = 3(y - 4)$
 $\Rightarrow y = 3y - 12$
 $\Rightarrow y - 3y = -12$
 $\Rightarrow -2y = -12$
 $\Rightarrow y = \frac{-12}{-2} = 6$

VI. Short answer type questions.

1. Find the solution to the system of equations $x = 2$ and $y = 5$ by drawing a graph.
2. Find the solution to the system of equations $x = -1$ and $y = 3$ by drawing a graph.
3. Draw the graph of the equation $5x + 4y + 20 = 0$. From the graph find the co-ordinate of the point when i) $x = 0$ ii) $y = -5$

4. Solve the system of equations by elimination method: $3x = y + 5$; $5x - y = 11$

Sol: $3x = y + 5$

$$\Rightarrow 3x - y = 5 \dots\dots\dots(1)$$

$$5x - y = 11 \dots\dots\dots(2)$$

Subtracting (1) - (2)

$$3x - y - (5x - y) = 5 - 11$$

$$\Rightarrow 3x - y - (5x - y) = 5 - 11$$

$$\Rightarrow 3x - y - 5x + y = -6$$

$$\Rightarrow -2x = -6 \Rightarrow x = -6/-2 = 3$$

Now substituting x value by 3 in equation (1)

$$3x - y = 5 \Rightarrow 3 \times 3 - y = 5$$

$$\Rightarrow -y = 5 - 9 = -4 \Rightarrow y = 4$$

$$(x, y) = (3, 4)$$

5. Solve the following pair of equations using formula: $49x + 51y = 499$; $51x + 49y = 501$

Sol: $49x + 51y = 499$; $51x + 49y = 501$

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow x = \frac{51 \times 501 - 49 \times 499}{49 \times 49 - 51 \times 51} = \frac{25551 - 24451}{2401 - 2601}$$

$$\Rightarrow x = \frac{1100}{-200} = -\frac{11}{2} = -5.5$$

$$y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow y = \frac{499 \times 51 - 501 \times 49}{49 \times 49 - 51 \times 51} = \frac{25449 - 24549}{2401 - 2601}$$

$$\Rightarrow y = \frac{900}{-200} = -\frac{9}{2} = -4.5$$

6. Find the two numbers whose sum is 75 and difference is 15.

Sol: Let the two numbers x and y.

$$\therefore x + y = 75 \dots\dots\dots(1)$$

$$x - y = 15 \dots\dots\dots(2)$$

Adding (1) and (2)

$$x + y + x - y = 75 + 15$$

$$\Rightarrow 2x = 90 \Rightarrow x = 90/2 = 45$$

Substituting x by 45 in equation (1) we get

$$45 + y = 75$$

$$\Rightarrow y = 75 - 45 = 30$$

Two numbers are 45 and 30

7. Find the value of α and β for which the following pair of linear equations has infinite number of solutions: $2x + 3y = 7$; $\alpha x + (\alpha + \beta)y = 28$

Sol: $2x + 3y = 7$; $\alpha x + (\alpha + \beta)y = 28$

If the pair of linear equation has infinite solutions then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\begin{array}{l|l} \Rightarrow \frac{2}{\alpha} = \frac{3}{(\alpha + \beta)} = \frac{7}{28} & \frac{3}{(8 + 7\beta)} = \frac{7}{28} \\ \Rightarrow \frac{2}{\alpha} = \frac{7}{28} \Rightarrow 7\alpha = 56 & \Rightarrow 56 + 7\beta = 84 \\ \Rightarrow \alpha = 56/7 = 8 & \Rightarrow 7\beta = 84 - 56 = 28 \\ & \Rightarrow \beta = 28/7 = 4 \end{array}$$

8. Solve the following pair of linear equations by the substitute method: $x + 2y = 2$; $x - 3y = 7$

Sol: $x + 2y = 2$

$$\Rightarrow x = 2 - 2y$$

Substituting x by $2 - 2y$ in $x - 3y = 7$

$$x - 3y = 7$$

$$\Rightarrow 2 - 2y - 3y = 7 \Rightarrow -5y = 7 - 2 = 5 \Rightarrow y = 5/-5 = -1$$

$$x = 2 - 2y$$

$$\Rightarrow x = 2 - 2 \times -1 = 2 + 2 = 4$$

$$(x, y) = (4, -1)$$

9. A man earns Rs. 600 per month more than his wife. One-tenth of the man's salary and 1/6th of the wife's salary amount to Rs. 1,500, which is saved every month. Find their incomes.

Sol: Let the man's salary = x and the lady's salary = y

A man earns Rs. 600 per month more than his wife.

$$\therefore x = y + 600$$

One-tenth of the man's salary and 1/6th of the wife's salary amount to Rs. 1,500

$$\frac{x}{10} + \frac{y}{6} = 1500$$

$$\Rightarrow \frac{3x + 5y}{30} = 1500$$

$$\Rightarrow 3x + 5y = 1500 \times 30 = 45000$$

$$\Rightarrow 3x + 5y = 45000$$

Substituting x by $y + 600$

$$\Rightarrow 3(y + 600) + 5y = 45000 \Rightarrow 3y + 1800 + 5y = 45000$$

$$\Rightarrow 8y = 45000 - 1800 = 43200 \Rightarrow y = 43200/8 = 5400$$

$$x = y + 600 = 5400 + 600 = 6000$$

The man's salary is Rs. 6000 and the lady's salary is Rs. 5400

10. $7x - 5y - 4 = 0$ is given. Write another linear equation, so that the lines represented by the pair are parallel to each other.

Sol: $7x - 5y - 4 = 0$

Parallel lines if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\therefore$ the other linear equation can be $14x - 10y + 3 = 0$

VII. Long answer type questions.

1. 10 students of Class-X took part in a mathematics quiz. If the number of girls is 4 more than the number of boys then, find the number of boys and the number of girls who took part in the quiz.

Sol: Let the number of girls = x

The number of boys = y

Number of girls is 4 more than the number of boys.

$$\therefore x = y + 4$$

Total students took part in a mathematics quiz = 10

$$\therefore x + y = 10 \quad (\text{substituting } x \text{ by } y + 4)$$

$$\Rightarrow y + 4 + y = 10 \Rightarrow 2y = 10 - 4 = 6$$

$$\Rightarrow y = 6/2 = 3$$

$$x = y + 4 \Rightarrow x = 3 + 4 = 7$$

Number of girls 7 and number of boys 3

2. 5 pencils and 7 pens together cost 50 whereas 7 pencils and 5 pens together cost 46. Find the cost of one pencil and that of one pen.

Sol: Let cost of a pencil = x

Cost of a pen = y

5 pencils and 7 pens together cost 50

$$\therefore 5x + 7y = 50 \dots\dots\dots (1)$$

7 pencils and 5 pens together cost 46

$$\therefore 7x + 5y = 46 \dots\dots\dots (2)$$

Equation (1) $\times 5$ - (2) $\times 7$

$$\text{Eq: (1)} \quad 25x + 35y = 250$$

$$\text{Eq: (2)} \quad 49x + 35y = 322$$

$$\Rightarrow -24x = -72$$

$$\Rightarrow x = -72/-24 = 3$$

Substituting x by 3 in equation (1) we get

$$5 \times 3 + 7y = 50$$

$$\Rightarrow 7y = 50 - 15 = 35 \Rightarrow y = 35/7 = 5$$

The cost of a pen is 5 rupees and the cost of a pencil is 3 rupees.

3. Half the perimeter of a rectangular garden, whose length is 4m more than its width, is 36m. Find the dimensions of the garden.

Sol: Let the length of the garden = x m.

Width of the garden = y m.

Half the perimeter of a rectangular garden is 36 m.

$$\therefore x + y = 36 \dots\dots\dots(1)$$

Length is 4 m more than its width.

$$\therefore x = y + 4$$

Substituting x by $y + 4$ in equation (1)

$$y + 4 + y = 36$$

$$\Rightarrow 2y = 36 - 4 = 32 \Rightarrow y = 32/2 = 16$$

$$x = y + 4$$

$$\Rightarrow x = 16 + 4 = 20$$

$\therefore$ Length of the rectangular garden is 20 m and width is 16 m.

4. In X class, if three students sit on each bench, one student will be left. If four students sit on each bench, one bench will be left. Find the number of students and the number of benches in that class.

Sol: Let the number of students = x

Number of benches = y

If three students sit on each bench, one student will be left.

$$\therefore x = 3y + 1$$

If four students sit on each bench, one bench will be left

$$\therefore x = 4y - 4$$

$$\therefore 3y + 1 = 4y - 4$$

$$\Rightarrow 3y - 4y = -4 - 1 \Rightarrow -y = -5 \Rightarrow y = 5$$

$$x = 4y - 4$$

$$\Rightarrow x = 4 \times 5 - 4 = 16$$

Number of students = 16 and number of benches = 5

5. The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to save 2000 per month, find their monthly income.

Sol: Let the two persons income are x and y

The ratio of incomes of two persons is 9 : 7

$$\therefore \frac{x}{y} = \frac{9}{7} \Rightarrow 7x = 9y \Rightarrow x = \frac{9y}{7}$$

The ratio of their expenditures is 4 : 3. And Each of them manages to save 2000 per month.

$$\therefore \frac{x-2000}{y-2000} = \frac{4}{3} \Rightarrow 7y - 3\left(\frac{9y}{7}\right) = 2000 \Rightarrow y = 14000$$

$$x = \frac{9y}{7} = \frac{9 \times 14000}{7} = 18000$$

Two persons incomes are Rs. 18000 and Rs. 16000.

6. The sum of a two digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there?

Sol: Let the tens place digit of the number = x

The unit place digit of the number = y

The number = 10x + y

The reverse number = 10y + x

The sum of a two digit number the number obtained by reversing the digits is 66.

$$\therefore 10x + y + 10y + x = 66$$

$$\Rightarrow 11x + 11y = 66$$

$$\Rightarrow x + y = 6 \dots\dots\dots (1)$$

Digits of the number differ by 2

$$\Rightarrow x - y = 2 \dots\dots\dots (2)$$

Equation (1) + (2)

$$x + y + x - y = 6 + 2$$

$$\Rightarrow 2x = 8 \Rightarrow x = 8/2 = 4$$

Substituting x by 4 in equation (2)

$$4 - y = 2$$

$$\Rightarrow -y = 2 - 4 = -2 \Rightarrow y = 2$$

$$\therefore \text{the number} = 10x + y = 10 \times 4 + 2 = 42$$

There are 2 such numbers. They are 42 and 24.

7. Two angles are complementary. The larger angle is 3° less than twice the measure of the smaller angle. Find the measure of each angle.

Sol: let the two complementary angles are x and y

x is larger angle and y is smaller angle.

The larger angle is 3° less than twice the measure of the smaller angle.

$$x = 2y - 3$$

$$x + y = 90 \quad (\text{x and y are complementary angles})$$

$$\Rightarrow 2y - 3 + y = 90 \quad (\text{substituting x by } 2y - 3)$$

$$\Rightarrow 3y = 90 + 3 = 93$$

$$\Rightarrow y = 93/3 = 31$$

$$x = 2y - 3 = 2 \times 31 - 3 = 59$$

Two angles are 59° and 31° .

8. An algebra textbook has a total of 1382 pages. It is broken up into two parts. The second part of the book has 64 pages more than the first part. How many pages are in each part of the book?

Sol: An algebra textbook has a total of 1382 pages.

It is broken up into two parts.

Let the first part contains pages = x

Second part contains pages = y

$$\therefore x + y = 1382 \dots\dots\dots(1)$$

The second part of the book has 64 pages more than the first part.

$$\therefore y = x + 64$$

Now substituting y by $x+64$ in equation (1)

$$x + x + 64 = 1382$$

$$\Rightarrow 2x = 1382 - 64 = 1318$$

$$\Rightarrow x = 1318/2 = 659$$

$$y = x + 64 = 659 + 64 = 723$$

9. A chemist has two solutions of hydrochloric acid in stock. One is 50% solution and the other is 80% solution. How much of each should be used to obtain 100ml of a 68% solution.

Sol: First solution contains 50% acid.

Second solution contains 80% acid.

Let x ml of 1st solution and y ml of second solution are added.

$$\text{Then } x + y = 100 \dots\dots\dots(1)$$

GOLD PACKAGE



I. Choose the correct answer.

1. Given that $5m - k = 4$ and $3m - 5k = -24$, the value of m is:

- a) $-\frac{2}{11}$ b) -2 c) 2 d) None of these

Sol: $5m - k = 4$ (1)

$3m - 5k = -24$ (2)

Equation (1) $\times 5 \Rightarrow 25m - 5k = 20$

$$\begin{array}{rcl} \text{Equation (2)} & \Rightarrow & 3m - 5k = -24 \\ & (-) & \underline{- \quad + \quad +} \\ & & 22m = 44 \\ & \Rightarrow & m = 44/22 = 2 \end{array}$$

The value of $m = 2$.

Ans: (C) 2

2. If the places of last two digits of a three digit number are interchanged, a new number greater than the original number by 27 is obtained. Find the difference between the last two digits of that number.

- a) 6 b) 3 c) 8 d) None of these

Sol: Let the hundred place digit = x ; Tens place digit = y , Unit place digit = z

The number = $100x + 10y + z$

Last two digits of the three digit number interchanged.

The new number = $100x + 10z + y$

The new number is greater than the original number by 27.

$$\therefore 100x + 10z + y = 100x + 10y + z + 27$$

$$\Rightarrow 100x + 10z + y - 100x - 10y - z = 27$$

$$\Rightarrow 9z - 9y = 27 \Rightarrow z - y = 27/9 = 3$$

The difference between last two digits is 3

Ans: (B) 3

3. Given that $\frac{3x-4}{4x-3} = 2$, find the value of x :

- a) 3 b) -3 c) -2 d) None of these

Sol: $\frac{3x-4}{4x-3} = 2$

$$\Rightarrow 3x - 4 = 2(4x - 3) \Rightarrow 3x - 4 = 8x - 6 \Rightarrow 3x - 8x = -6 + 4$$

$$\Rightarrow -5x = -2 \Rightarrow x = \frac{2}{5}$$

Ans: (D) None of these

4. A boy was asked to multiply a certain number by 53. Instead he divided it by 53 and got his answer to be less than the correct one by 5616. The number to be multiplied was:
- a) 53 b) 106 c) 159 d) None of these

Sol: Let the certain number = x

A boy was asked to multiply a certain number by 53.

The correct result = $53x$

Instead he divided it by 53

The wrong result = $x/53$

He got his answer to be less than the correct one by 5616

$$\Rightarrow 53x = \frac{x}{53} + 5616 \Rightarrow 53x = \frac{x + 297648}{53}$$

$$\Rightarrow 53x \times 53 = x + 297648 \Rightarrow 2809x - x = 297648$$

$$\Rightarrow 2808x = 297648 \Rightarrow x = \frac{297648}{2808} = 106$$

The number to be multiplied was 106

Ans: (B) 106

5. if $7p + 3 = 3(5 - p)$, then $p =$

- a) 3 b) $2\frac{1}{4}$ c) $1\frac{1}{5}$ d) None of these

Sol: $7p + 3 = 3(5 - p)$

$$\Rightarrow 7p + 3 = 15 - 3p \Rightarrow 7p + 3p = 15 - 3$$

$$\Rightarrow 10p = 12 \Rightarrow p = 12/10 = 1\frac{1}{5}$$

Ans: (C) $1\frac{1}{5}$

6. What are the number of integer solutions of the equation $7x + 3y = 123$, for $x, y > 0$?

- a) 3 b) 4 c) 5 d) None of these

Sol: $7x + 3y = 123$

$$\Rightarrow 7x = 123 - 3y$$

$$\Rightarrow x = \frac{123 - 3y}{7}$$

Ans: (C) 5

7) The difference between two numbers is 5 and the difference of their square is 65. The larger number is:

- a) 9 b) 10 c) 11 d) None of these

Sol: Let the larger number = x and the smaller number = y

The difference between two numbers is 5

$$\Rightarrow x - y = 5 \dots\dots\dots (1)$$

The difference of their square is 65

$$\Rightarrow x^2 - y^2 = 65 \Rightarrow (x + y)(x - y) = 65$$

$$\Rightarrow x + y = 65/5 = 13 \dots\dots\dots (2)$$

By adding (1) and (2) we get

$$x - y + x + y = 5 + 13$$

$$\Rightarrow 2x = 18 \Rightarrow x = 18/2 = 9$$

The larger number is 9

Ans: (A) 9

8. The sum of ages of a father and son is 45 years. Five years ago, the product of their ages was 4 times the age of the father at that time. The present age of the father is:

- a) 30 years b) 31 years c) 37 years d) None of these

Sol: Let the present age of father = x

The sum of ages of a father and son is 45 years.

$$\therefore \text{Present age of son} = 45 - x$$

Five years ago, the product of their ages was 4 times the age of the father at that time.

$$\Rightarrow (x - 5)(45 - x - 5) = 4(x - 5)$$

$$\Rightarrow (x - 5)(40 - x) = 4x - 20$$

$$\Rightarrow 40x - x^2 - 200 + 5x = 4x - 20$$

$$\Rightarrow -x^2 + 45x - 4x - 200 + 20 = 0$$

$$\Rightarrow -x^2 + 41x - 180 = 0$$

$$\Rightarrow x^2 - 41x + 180 = 0$$

$$\Rightarrow x^2 - 36x - 5x + 180 = 0$$

$$\Rightarrow x(x - 36) - 5(x - 36) = 0$$

$$\Rightarrow (x - 36)(x - 5) = 0$$

$$\Rightarrow x = 36 \text{ or } x = 5 \text{ (Not possible)}$$

$$\therefore x = 36$$

Ans: (D) None of these

9) Find three consecutive integers such that four times the first plus one-half the second minus twice the third is equal to 24

- a) 11, 12 and 13 b) 13, 14 and 15 c) 12, 13 and 14 d) None of these

Sol: Let the three consecutive integers are $x, x+1, x+2$

Four times the first plus one-half the second minus twice the third is equal to 24

$$\Rightarrow 4x + \frac{1}{2}(x+1) - 2(x+2) = 24$$

$$\Rightarrow 4x + \frac{x+1}{2} - 2x - 4 = 24 \Rightarrow 2x + \frac{x+1}{2} = 24 + 4 = 28$$

$$\Rightarrow \frac{4x + x + 1}{2} = 28 \Rightarrow 4x + x + 1 = 28 \times 2 = 56$$

$$\Rightarrow 5x = 56 - 1 = 55 \Rightarrow x = \frac{55}{5} = 11$$

$$x + 1 = 11 + 1 = 12$$

$$x + 2 = 11 + 2 = 13$$

Ans: (A) 11, 12 and 13

10) Sudha is 6 years older than Radha. Five years ago, Sudha was thrice as old as Radha. Find the present age of Sudha.

- a) 8 years b) 14 years c) 16 years d) None of these

Sol: Let Sudha's present age = x

Radha's present age = y

Sudha is 6 years older than Radha.

$$\Rightarrow x = y + 6 \Rightarrow x - y = 6 \dots\dots\dots(1)$$

Five years ago, Sudha was thrice as old as Radha.

$$\Rightarrow x - 5 = 3(y - 5)$$

$$\Rightarrow x - 5 = 3y - 15$$

$$\Rightarrow x - 3y = -15 + 5 = -10 \dots\dots\dots(2)$$

$$(1) - (2)$$

$$x - y - (x - 3y) = 6 - (-10)$$

$$\Rightarrow x - y - x + 3y = 16$$

$$\Rightarrow 2y = 16 \Rightarrow y = 16/2 = 8$$

Substituting y by 8 in equation (1)

$$x - 8 = 6 \Rightarrow x = 6 + 8 = 14$$

Sudha's age = 14

Ans: (B) 14 years

II. Answer the following questions.

1. Solve $3 + \frac{x}{4} = \frac{1}{2}\left(4 - \frac{x}{3}\right) - \frac{5}{6} + \frac{1}{3}\left(11 - \frac{x}{2}\right)$

Sol:

$$\begin{aligned} 3 + \frac{x}{4} &= \frac{1}{2}\left(4 - \frac{x}{3}\right) - \frac{5}{6} + \frac{1}{3}\left(11 - \frac{x}{2}\right) \\ \Rightarrow 3 + \frac{x}{4} &= \frac{1}{2}\left(\frac{12-x}{3}\right) - \frac{5}{6} + \frac{1}{3}\left(\frac{22-x}{2}\right) \\ \Rightarrow 3 + \frac{x}{4} &= \frac{12-x}{6} - \frac{5}{6} + \frac{22-x}{6} \\ \Rightarrow \frac{12+x}{4} &= \frac{12-x-5+22-x}{6} \\ \Rightarrow 6(12+x) &= 4(29-2x) \Rightarrow 72+6x = 116-8x \\ \Rightarrow 6x+8x &= 116-72 \Rightarrow 14x = 44 \\ \Rightarrow x &= \frac{44}{14} = \frac{22}{7} \end{aligned}$$

2. Solve: $\frac{x+1}{2} + \frac{y-1}{3} = 8$ and $\frac{x-1}{3} + \frac{y+1}{2} = 9$

Sol:

$$\frac{x+1}{2} + \frac{y-1}{3} = 8$$

$$\Rightarrow \frac{3(x+1) + 2(y-1)}{6} = 8$$

$$\Rightarrow 3(x+1) + 2(y-1) = 8 \times 6$$

$$\Rightarrow 3x + 3 + 2y - 2 = 48$$

$$\Rightarrow 3x + 2y = 48 - 1$$

$$\Rightarrow 3x + 2y = 47 \dots\dots\dots(1)$$

$$\frac{x-1}{3} + \frac{y+1}{2} = 9$$

$$\Rightarrow \frac{2(x-1) + 3(y+1)}{6} = 9$$

$$\Rightarrow 2x - 2 + 3y + 3 = 9 \times 6$$

$$\Rightarrow 2x + 3y + 1 = 54$$

$$\Rightarrow 2x + 3y = 54 - 1$$

$$\Rightarrow 2x + 3y = 53 \dots\dots\dots(2)$$

$$\text{Equation (1)} \times 2 \Rightarrow 6x + 4y = 94$$

$$\text{Equation (2)} \times 3 \Rightarrow 6x + 9y = 159$$

$$\begin{array}{r} (-) \quad - \quad - \quad - \\ \hline -5y = -65 \\ \Rightarrow y = -65/-5 = 13 \end{array}$$

Substituting y by 13 in equation (1)

$$3x + 2 \times 13 = 47$$

$$\Rightarrow 3x = 47 - 26 = 21$$

$$\Rightarrow x = 21/3 = 7$$

$$x = 7 \text{ and } y = 13$$

3. Solve: $2x - 3y = 1$ and $3x - 4y = 1$ graphically.

Sol: First equation: $2x - 3y = 1$

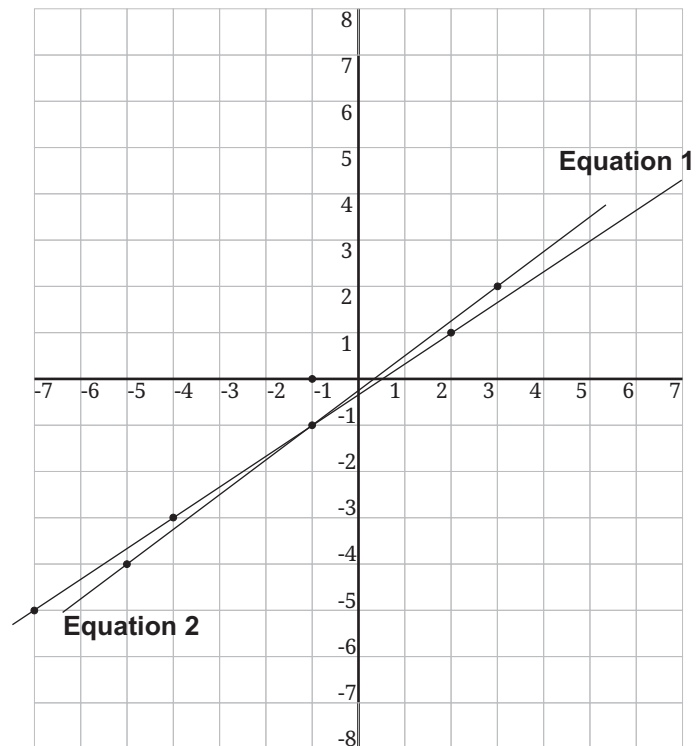
$$\Rightarrow y = \frac{1-2x}{-3}$$

x	2	-4	-7
$y = \frac{1-2x}{-3}$	1	-3	-5
(x, y)	(2, 1)	(-4, -3)	(-7, -5)

Second equation: $3x - 4y = 1$

$$\Rightarrow y = \frac{1-3x}{-4}$$

x	-1	3	-5
$y = \frac{1-3x}{-4}$	-1	2	-4
(x, y)	(-1, -1)	(3, 2)	(-5, -4)



The solution is $(-1, -1)$

4. Solve: $\frac{5}{x+y} - \frac{2}{x-y} = -1$ and $\frac{15}{x+y} + \frac{7}{x-y} = 10$

Sol: Let $a = \frac{1}{x+y}$ and $b = \frac{1}{x-y}$

$$\frac{5}{x+y} - \frac{2}{x-y} = -1$$

$$\Rightarrow 5a - 2b = -1 \dots\dots\dots(1)$$

$$\frac{15}{x+y} + \frac{7}{x-y} = 10$$

$$\Rightarrow 15a + 7b = 10 \dots\dots\dots(2)$$

$$\text{Equation (1)} \times 3 \Rightarrow 15a - 6b = -3$$

$$\text{Equation (2)} \Rightarrow 15a + 7b = 10$$

$$\begin{array}{r} (-) - \quad - \quad - \\ \hline \end{array}$$

$$\Rightarrow -13b = -13$$

$$\Rightarrow b = -13/-13 = 1$$

Substituting b by 1 in equation (1)

$$\Rightarrow 5a - 2 \times 1 = -1$$

$$\Rightarrow 5a = -1 + 2 = 1$$

$$\Rightarrow a = 1/5$$

$$\therefore \frac{1}{x+y} = \frac{1}{5} \Rightarrow x+y = 5 \dots\dots\dots(3)$$

$$\therefore \frac{1}{x-y} = 1 \Rightarrow x-y = 1 \dots\dots\dots(4)$$

Equation (3) + (4)

$$x + y + x - y = 5 + 1$$

$$\Rightarrow 2x = 6 \Rightarrow x = 6/2 = 3$$

Substituting x by 3 in equation (3)

$$\Rightarrow 3 + y = 5 \Rightarrow y = 5 - 3 = 2$$

So $x = 3$ and $y = 2$

5. Solve: $\sqrt{3}x + \sqrt{2}y = \sqrt{3}$ and $\sqrt{5}x + \sqrt{3}y = \sqrt{3}$

Sol: $\sqrt{3}x + \sqrt{2}y = \sqrt{3}$

multiply both side by $\sqrt{3}$

$$\Rightarrow 3x + \sqrt{6}y = 3 \dots \dots \dots (1)$$

$$\sqrt{5}x + \sqrt{3}y = \sqrt{3}$$

Multiplying both side by $\sqrt{2}$

$$\Rightarrow \sqrt{10}x + \sqrt{6}y = \sqrt{6} \dots \dots \dots (2)$$

Equation (1) - Equation (2)

$$\Rightarrow 3x + \sqrt{6}y - (\sqrt{10}x + \sqrt{6}y) = 3 - \sqrt{6}$$

$$\Rightarrow 3x + \sqrt{6}y - \sqrt{10}x - \sqrt{6}y = 3 - \sqrt{6}$$

$$\Rightarrow 3x - \sqrt{10}x = 3 - \sqrt{6}$$

$$\Rightarrow x(3 - \sqrt{10}) = 3 - \sqrt{6}$$

$$\Rightarrow x = \frac{3 - \sqrt{6}}{3 - \sqrt{10}}$$

$$\sqrt{3}x + \sqrt{2}y = \sqrt{3}$$

multiply both side by $\sqrt{5}$

$$\Rightarrow \sqrt{15}x + \sqrt{10}y = \sqrt{15} \dots \dots \dots (3)$$

$$\sqrt{5}x + \sqrt{3}y = \sqrt{3}$$

Multiplying both side by $\sqrt{3}$

$$\Rightarrow \sqrt{15}x + 3y = 3 \dots \dots \dots (4)$$

Equation (3) - Equation (4)

$$\Rightarrow \sqrt{15}x + \sqrt{10}y - (\sqrt{15}x + 3y) = \sqrt{15} - 3$$

$$\Rightarrow \sqrt{15}x + \sqrt{10}y - \sqrt{15}x - 3y = \sqrt{15} - 3$$

$$\Rightarrow \sqrt{10}y - 3y = \sqrt{15} - 3$$

$$\Rightarrow y(\sqrt{10} - 3) = \sqrt{15} - 3$$

$$\Rightarrow y = \frac{\sqrt{15} - 3}{(\sqrt{10} - 3)}$$

6. The sum of the digits of a two digit number is 12. The number obtained by interchanging the two digits exceeds the given number by 18. Find the number.

Sol: Let the tens place digit = x and unit place digit = y

The sum of the digits of a two digit number is 12.

$$\Rightarrow x + y = 12 \dots\dots\dots(1)$$

$\therefore$ The two digit number = $10x + y$

The number obtained by interchanging the two digits exceeds the given number by 18.

The number after interchanging two digits = $10y + x$

$$\therefore 10y + x = 10x + y + 18$$

$$\Rightarrow 10y + x - 10x - y = 18 \Rightarrow 9x - 9y = 18 \Rightarrow 9(x - y) = 18$$

$$\Rightarrow x - y = 18/9 = 2 \dots\dots\dots(2)$$

Equation (1) + Equation (2)

$$x + y + x - y = 12 + 2$$

$$\Rightarrow 2x = 14 \Rightarrow x = 14/2 = 7$$

Substituting x by 7 in equation (1)

$$7 + y = 12 \Rightarrow y = 12 - 7 = 5$$

So x = 7 and y = 5

7. The length and width of a rectangle is 4x cm and y cm respectively. If its length is 3 cm longer than its width, write an equation involving x and y.

Sol: $4x = y + 3$

$$\Rightarrow 4x - y = 3$$

8. For what value of a, the system of linear equations $ax + 3y = a - 3$, $12x + ay = a$ has no solution.

Sol: No solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\frac{a}{12} = \frac{3}{a} \neq \frac{a-3}{a}$$

$$\Rightarrow a^2 = 3 \times 6 = 36$$

$$\Rightarrow a = \sqrt{36} = \pm 6$$

9. Solve the system of equations . $\frac{bx}{a} - \frac{ay}{b} + a + b = 0$ and $bx - ay + 2ab = 0$

Sol: $bx - ay + 2ab = 0$

$$\Rightarrow bx = ay - 2ab$$

$$\Rightarrow x = \frac{ay - 2ab}{b}$$

$$\frac{bx}{a} - \frac{ay}{b} + a + b = 0$$

$$\Rightarrow \frac{b^2x - a^2y + a^2b + ab^2}{ab} = 0$$

$$\Rightarrow b^2x - a^2y + a^2b + ab^2 = 0$$

Substituting x by $\frac{ay - 2ab}{b}$

$$\Rightarrow b^2\left(\frac{ay - 2ab}{b}\right) - a^2y + a^2b + ab^2 = 0$$

$$\Rightarrow aby - 2ab^2 - a^2y + a^2b + ab^2 = 0$$

$$\Rightarrow aby - a^2y = ab^2 - a^2b$$

$$\Rightarrow y(ab - a^2) = b(ab - a^2)$$

$$\Rightarrow y = b$$

10. Solve the following equation graphically. $2x + 3y = 13$ and $x - y = 1$

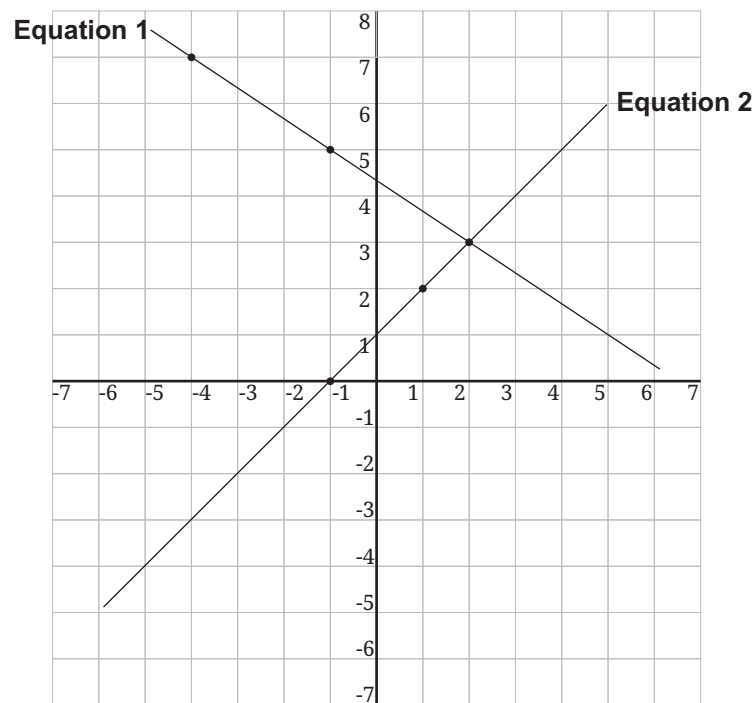
Sol: $2x + 3y = 13$

$$x - y = 1 \Rightarrow 1 + x$$

$$\Rightarrow y = \frac{13 - 2x}{3}$$

x	2	-1	-4
$\Rightarrow y = \frac{13 - 2x}{3}$	3	5	7
(x, y)	(2, 3)	(-1, 5)	(-4, 7)

x	-1	1	2
$y = 1 + x$	0	2	3
(x, y)	(-1, 0)	(1, 2)	(2, 3)



Solution:
(x, y) = (2, 3)

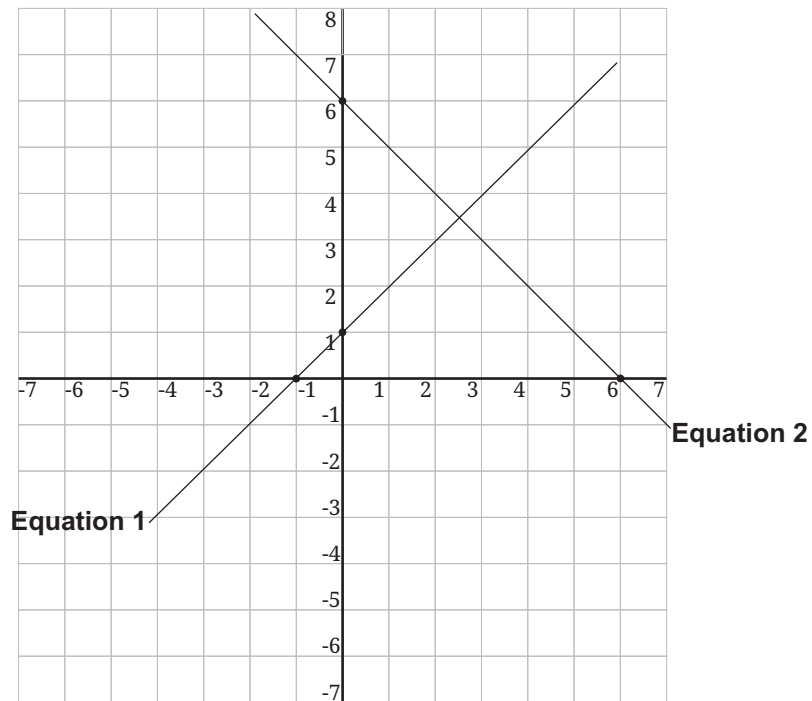
11. Draw the graph of $x - y + 1 = 0$ and $2x + 2y - 12 = 0$ then calculate the area bounded by these lines and the x-axis.

Sol: $x - y + 1 = 0 \Rightarrow y = 1 + x$

$$2x + 2y - 12 = 0 \Rightarrow y = \frac{12 - 2x}{2}$$

x	0	-1
$y = 1 + x$	1	0
(x, y)	(0, 1)	(-1, 0)

x	0	6
$\Rightarrow y = \frac{12 - 2x}{2}$	6	0
(x, y)	(0, 6)	(6, 0)



In the above triangle

$h = 3.5$ units

$b = 7$ units

Area = $\frac{1}{2} \times b \times h = \frac{1}{2} \times 3.5 \times 7 = 12.25$ sq. units.

EXERCISE - 3**QUADRATIC EQUATIONS****I. Choose the correct answer.**

1. If $x^2 + 5y^2 + z^2 = 2y(2x + z)$ then which of the following statement is true.

- a) $x = 2y$ b) $x = 2z$ c) $2x = z$ d) None of these

Sol: $x^2 + 5y^2 + z^2 = 2y(2x + z)$
 $\Rightarrow [x^2 + 4y^2 - 2(2y)x] + (y^2 + z^2 - yz) = 0$
 $\Rightarrow (x - 2y)^2 + (y - z)^2 = 0$
 $\Rightarrow (x - 2y)^2 = 0 \text{ \& } (y - z)^2 = 0$
 $\Rightarrow x = 2y \text{ \& } y = z \Rightarrow x = 2z$

Ans: (A) $x = 2y$, (B) $x = 2z$

2. The roots of the quadratic equation $4x^2 - 4x + 1 = 0$ are

- a) not real b) real and equal c) real and irrational d) None of these

Sol: $D = b^2 - 4ac$
 $\Rightarrow D = (-4)^2 - 4 \times 4 \times 1 = 16 - 16 = 0$
 $\Rightarrow D = 0$ (Roots are real and equal)

Ans: (B) real and equal

3. The roots of $2x^2 - 7x + 3 = 0$ are:

- a) real and equal b) real but not equal c) not real d) None of these

Sol: $D = b^2 - 4ac$
 $\Rightarrow D = (-7)^2 - 4 \times 2 \times 3 = 49 - 24 = 25$
 $\Rightarrow D > 0$ (Roots are real and unequal)

Ans: (B) real but not equal

4) The number of roots of quadratic equation is

- a) 1 b) 2 c) 3 d) None of these

Ans: (B) 2

5) The equation whose roots are $3 + 2$ and $3 - 2$ is

- a) $x^2 - 6x - 1 = 0$ b) $x^2 - 6x + 1 = 0$ c) $x^2 + 6x - 1 = 0$ d) None of these

Sol: $\alpha = 3 + 2 = 5$;
 $\beta = 3 - 2 = 1$
 The equation: $x^2 - (\alpha + \beta)x + (\alpha\beta) = 0$
 $\Rightarrow x^2 - (5 + 1)x + (5 \times 1) = 0$
 $\Rightarrow x^2 - 6x + 5 = 0$

Ans: (D) None of these

6) If one root of $px^2 - 14x + 8 = 0$ is six times the other, then the value of p is:

a) 1

b) 2

c) 3

d) None of these

Sol: The given quadratic equation is $px^2 - 14x + 8 = 0$

Let the root be α then other root will be 6α

Sum of the roots = $-b/a$

$$\Rightarrow \alpha + 6\alpha = \frac{14}{p} \Rightarrow 7\alpha = \frac{14}{p} \Rightarrow \alpha = \frac{14}{p} \times \frac{1}{7} = \frac{2}{p}$$

Product of the roots = c/a

$$\Rightarrow \alpha \times 6\alpha = 8/p \Rightarrow 6\alpha^2 = 8/p$$

$$\Rightarrow 6\left(\frac{2}{p}\right)^2 = \frac{8}{p} \Rightarrow \frac{24}{p^2} = \frac{8}{p} \Rightarrow 24p = 8p^2$$

$$\Rightarrow \frac{p}{p^2} \frac{8}{24} = \frac{1}{p} = \frac{1}{3} \Rightarrow p = 3$$

Ans: (C) 3

7) If α, β are the roots of $ax^2 + bx + c = 0$, then the value of $\alpha\beta^2 + \alpha^2\beta + \alpha\beta$ is:

a) $\frac{ac-bc}{a^2}$

b) $\frac{bc-ac}{a^2}$

c) $\frac{ac-bc}{b^2}$

d) None of these

Sol: $\alpha + \beta = \frac{-b}{a}$

$$\alpha\beta = \frac{c}{a}$$

$$\alpha\beta^2 + \alpha^2\beta + \alpha\beta = \alpha\beta(\beta + \alpha) + \alpha\beta$$

$$= \frac{-b}{a} \left(\frac{c}{a} \right) + \frac{c}{a} = \frac{-bc}{a^2} + \frac{c}{a} = \frac{-bc + ac}{a^2}$$

Ans: (A) $\frac{ac-bc}{a^2}$

8) If α, β are the roots of $ax^2 + bx + c = 0$, then the value of $\alpha^2 + \beta^2$ is:

a) $\frac{c^2}{a^2}$

b) $\frac{c^3}{a^3}$

c) $\frac{3abc-b^3}{c^3}$

d) None of these

Sol: $\alpha + \beta = \frac{-b}{a}$

$$\alpha\beta = \frac{c}{a}$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= \left(\frac{-b}{a} \right)^2 - 2 \times \frac{c}{a} = \frac{b^2}{a^2} - \frac{2c}{a} = \frac{b^2 - 2ac}{a^2}$$

Ans: (D) None of these

- 9) If $x^2 + px - 444p = 0$ has integral roots where p is a prime number, then the value of p is:
 a) 2 b) 3 c) 31 d) None of these

Sol: Given, $x^2 + px - 444p = 0$

$$x = -p \pm \sqrt{\frac{p^2 - 4 \times 1 \times -444p}{2 \times 1}}$$

Since $p = 2$ does not give the integral roots, so D must be a perfect square of an odd integer, i.e.,

$$D^2 = p^2 + 1776p = p(p + 1776)$$

Since D is perfect square, hence $p + 1776$ must be a multiple of p , i.e., 1776 must be a multiple of p .

Now, $1776 = 2^4 \times 3 \times 37$ hence $p = 2$ or 3 or 37 .

(i) If $p = 2$, then $p(p + 1776) = 2(2 + 1776) = 3556 = 4 \times 7 \times 127$, which is not a perfect square.

(ii) If $p = 3$, then $p(p + 1776) = 3(3 + 1776) = 5337$, which is not a perfect square as its last digit is 7.

(iii) If $p = 37$, then $p(p + 1776) = 37(37 + 1776) = 37^2 \times 7^2$, which is odd.

Hence, $p = 37$.

Ans: (D) None of these

- 10) If α and β are the roots of the equation $x^2 - x + 1 = 0$ the value of $\alpha^{2009} + \beta^{2009} =$
 a) 1 b) -1 d) -2 c) None of these

Sol: $x^3 + 1 = (x + 1)(x^2 - x + 1)$

So, since α is a root of $x^2 - x + 1 = 0$ we have,

$$\alpha^3 + 1 = (\alpha + 1)(\alpha^2 - \alpha + 1) = (\alpha + 1)(0) = 0$$

$$\text{So, } \alpha^3 = -1$$

$$\text{Similarly: } \beta^3 = -1$$

$$\text{Also: } x^2 - x + 1 = (x - \alpha)(x - \beta) = x^2 - (\alpha + \beta)x + \alpha\beta$$

$$\text{So: } \alpha + \beta = 1 \text{ and } \alpha\beta = 1$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 1 - 2 = -1$$

$$\text{So, } \alpha^{2009} + \beta^{2009} = \alpha^{3 \cdot 669 + 2} + \beta^{3 \cdot 669 + 2}$$

$$= (\alpha^3)^{669} \cdot \alpha^2 + (\beta^3)^{669} \cdot \beta^2 = (-1)^{669} \cdot (\alpha^2 + \beta^2) = (-1)(-1) = 1$$

Ans: (A) 1

II. Say True or False.

1. In the equation $ax^2 + bx + c = 0$, if $a = 0$ then the equation is called cubic equation.

Ans: False

2. $(px + q)(rx + s) = 0$ is a quadratic equation.

Sol: $(px + q)(rx + s) = 0$

$$\Rightarrow prx^2 + psx + qrx + qs = 0$$

$$\Rightarrow prx^2 + (ps + qr)x + qs = 0 \text{ (This is a quadratic equation)}$$

Ans: True

3. The roots of the equation $3x^2 + 2x + 5 = 0$ are real numbers.

Sol: $b^2 - 4ac = 2^2 - 4 \times 3 \times 5 = 4 - 60 = -56$

$$b^2 - 4ac < 0 \text{ (roots are imaginary)}$$

Ans: False

4. The roots of the equation $x^2 - 9 = 0$ are real numbers and equal.

Ans: False

5. If the discriminant of an equation is a perfect square, the roots of the equation are unreal.

Ans: False

6. Sum of the roots of an equation $ax^2 + bx + c = 0$ is $-b$ if $a = 0$.

Ans: False

7. The product of the roots of an equation $ax^2 + bx + c = 0$ is c if $a = 0$.

Ans: False

8. An quadratic equation is of $x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$ form.

Ans: True

9. The roots of $x^2 - x + 1 = 0$ are imaginary.

Sol: $b^2 - 4ac = 1^2 - 4 \times 1 \times 1 = 1 - 4 = -3$

$$b^2 - 4ac < 0 \text{ (roots are imaginary)}$$

Ans: True

10. If $b^2 - 4ac = 0$ then the roots of $ax^2 + bx + c = 0$ are irrational and equal.

Ans: False

III. Match the following.

1. $\alpha^2 + \beta^2$ [**E**] a) $\frac{b^2 - 2ac}{c^2}$
2. $\alpha^3 + \beta^3$ [**D**] b) $\frac{\sqrt{b^2 - 4ac}}{|a|}$
3. $\alpha - \beta$ [**B**] c) $\frac{-b}{c}$
4. $\frac{1}{\alpha} + \frac{1}{\beta}$ [**C**] d) $\frac{3abc - b^3}{a^3}$
5. $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ [**A**] e) $\frac{b^2 - 2ac}{a^2}$

Sol:

1. $\alpha^2 + \beta^2$

$$\alpha + \beta = \frac{-b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= \left(\frac{-b}{a}\right)^2 - 2 \times \frac{c}{a} = \frac{b^2}{a^2} - \frac{2c}{a} = \frac{b^2 - 2ac}{a^2}$$

3. $\alpha - \beta$

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}; \quad \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \alpha - \beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a} - \left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$$

$$\Rightarrow \alpha - \beta = \frac{-b + \sqrt{b^2 - 4ac} + b + \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \alpha - \beta = \frac{2\sqrt{b^2 - 4ac}}{2a} = \frac{\sqrt{b^2 - 4ac}}{a}$$

2. $\alpha^3 + \beta^3$

$$\alpha + \beta = \frac{-b}{a}; \quad \alpha\beta = \frac{c}{a}; \quad \alpha^3\beta^3 = \frac{c^3}{a^3}$$

$$(\alpha + \beta)^3 = \alpha^3 + \beta^3 + 3 \times \alpha\beta(\alpha + \beta)$$

$$\Rightarrow \left(\frac{-b}{a}\right)^3 = \alpha^3 + \beta^3 + 3 \times \frac{c}{a} \times \frac{-b}{a}$$

$$\Rightarrow \frac{-b^3}{a^3} = \alpha^3 + \beta^3 - \frac{3bc}{a^2}$$

$$\Rightarrow \alpha^3 + \beta^3 = \frac{-b^3}{a^3} + \frac{3bc}{a^2} = \frac{-b^3 + 3abc}{a^3}$$

4. $\frac{1}{\alpha} + \frac{1}{\beta}$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{-b}{a} \times \frac{a}{c} = \frac{-b}{c}$$

5. $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$$\alpha + \beta = \frac{-b}{a}; \quad \alpha\beta = \frac{c}{a}; \quad \alpha^2\beta^2 = \frac{c^2}{a^2}$$

$$\begin{aligned} \frac{1}{\alpha^2} + \frac{1}{\beta^2} &= \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2} = \frac{b^2 - 2ac}{a^2} \times \frac{a^2}{c^2} \\ &= \frac{b^2 - 2ac}{c^2} \end{aligned}$$

IV. More than one option correct:

1) If α is a root of the equation $2x(2x + 1) = 1$, then the other root is:

a) $3\alpha^2 + 4\alpha$

b) $4\alpha^3 - 3\alpha$

c) $-2\alpha(\alpha + 1)$

d) None of these

Sol: Given: $2x(2x + 1) = 1$ i.e. $4x^2 + 2x - 1 = 0$

Let the other root be α, β

$$2\alpha(2\alpha + 1) = 1 \Rightarrow \frac{1}{2} = \alpha(2\alpha + 1) \dots \dots \dots (1)$$

$$\Rightarrow \alpha + \beta = \frac{-2}{4} = -\frac{1}{2}$$

$$\Rightarrow \beta = -\frac{1}{2} - \alpha = -\alpha(2\alpha + 1) - \alpha = -2\alpha^2 - \alpha - \alpha = -2\alpha^2 - 2\alpha = -2\alpha(\alpha + 1)$$

Ans: $-2\alpha(\alpha + 1)$

2) The value of m so that the equations $3x^2 - 2mx - 4 = 0$ and $x(x - 4m) + 2 = 0$ may have a common roots is

a) $\frac{1}{\sqrt{2}}$

b) $-\frac{1}{\sqrt{2}}$

c) $\frac{1}{2}$

d) None of these

Sol: $3x^2 - 2mx - 4 = 0 \dots \dots \dots (1)$

$x(x - 4m) + 2 = 0 \Rightarrow x^2 - 4mx + 2 = 0 \dots \dots \dots (2)$

Both have a common roots

Let the common root is α

Then $3\alpha^2 - 2m\alpha - 4 = 0$ and $\alpha^2 - 4m\alpha + 2 = 0$

By cross-multiplication

$$\frac{\alpha^2}{\begin{vmatrix} -2m & -4 \\ -4m & 2 \end{vmatrix}} = \frac{\alpha}{\begin{vmatrix} -4 & 3 \\ 2 & 1 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 3 & -2m \\ 1 & -4m \end{vmatrix}}$$

$$\Rightarrow \frac{\alpha^2}{-4m - 16m} = \frac{\alpha}{-4 - 6} = \frac{1}{-12m + 2m}$$

$$\Rightarrow \frac{\alpha^2}{-20m} = \frac{\alpha}{-10} = \frac{1}{-10m}$$

$$\Rightarrow \frac{\alpha^2}{-2m} = \frac{\alpha}{-1} = \frac{1}{-m}$$

for the first two terms $\alpha = 2m$ and for the last two terms $\alpha = \frac{1}{m}$

$$\therefore 2m = \frac{1}{m} \Rightarrow 2m^2 = 1 \Rightarrow m = \frac{1}{\sqrt{2}}$$

Ans: (A) $\frac{1}{\sqrt{2}}$

3) If the equation $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ have a common root then

a) $a = b$

b) $a + b = 0$

c) $a + b = -1$

d) None of these

Sol:

$$x^2 + ax + b = 0$$

$$x^2 + bx + a = 0$$

Only one root is common: let β be the common root then $\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}^2 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$

$$\Rightarrow \begin{vmatrix} 1 & b \\ 1 & a \end{vmatrix}^2 = \begin{vmatrix} 1 & a \\ 1 & b \end{vmatrix} \begin{vmatrix} a & b \\ b & a \end{vmatrix}$$

$$\Rightarrow (a - b)^2 = (b - a)(a^2 - b^2)$$

$$\Rightarrow (a - b)^2 = -(a - b)(a - b)(a + b)$$

$$\Rightarrow (a - b)^2 = -(a - b)^2 (a + b)$$

$$\Rightarrow (a - b)^2 + (a - b)^2 (a + b) = 0$$

$$\Rightarrow (a - b)^2 [1 + a + b] = 0$$

$$\Rightarrow (a - b)^2 = 0$$

$$\Rightarrow a = b$$

(or)

$$1 + a + b = 0$$

$$\Rightarrow a + b = -1$$

Ans: (A) $a = b$, (C) $a + b = -1$

4) If $b^2 = 4ac$ for the equation $ax^4 + bx^2 + c = 0$ then all the roots of the equation will be real if:

a) $b > 0, a < 0, c > 0$

b) $b < 0, a > 0, c > 0$

c) $b > 0, a > 0, c > 0$

d) All of these

Ans: (D) All of these

5) If $x = 2 + \sqrt{2 + \sqrt{2 + \sqrt{2}}}$, then $x =$

a) 1

b) 4

c) 2

d) None of these

Sol:

$$x = 2 + \sqrt{2 + \sqrt{2 + \sqrt{2}}} \dots\dots\dots$$

$$\Rightarrow x^2 = (2 + \sqrt{2 + \sqrt{2 + \sqrt{2}}} \dots\dots\dots)^2$$

$$\Rightarrow x^2 = 4 + 2 + \sqrt{2 + \sqrt{2 + \sqrt{2}}} \dots\dots\dots + 2 \times 2 \times \sqrt{2 + \sqrt{2 + \sqrt{2}}} \dots\dots\dots$$

$$\Rightarrow x^2 = 6 + x + 4x$$

$$\Rightarrow x^2 - 5x - 6 = 0$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{25 + 24}}{2} = \frac{5 \pm \sqrt{49}}{2}$$

$$x = \frac{5 + 7}{2} = 6$$

$$(or) x = \frac{5 - 7}{2} = -1$$

Ans: (D) None of these

V. Reason Type Questions

Instruction: The questions given below, consists of two statements viz Statement-1 (assertion) and Statement-2 (reason). Use the following key to choose appropriate answer.

- (a) Both statements are correct and Statement-2 is the correct explanation of Statement-1.
- (b) Both statements are correct but Statement-2 is not correct explanation of Statement-1.
- (c) Statement-1 is true & Statement-2 is false.
- (d) Statement-1 is false & Statement-2 is true.
- (e) Both the statements are false.

1) **Statement-1** : The roots of $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$ are real.

Statement-2 : A quadratic equation with non-negative discriminate has real roots.

Sol: Statement - 1: $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$
 $\Rightarrow x^2 - bx - ax + ab + x^2 - cx - bx + bc + x^2 - ax - cx - ca = 0$
 $\Rightarrow 3x^2 - 2ax - 2bx - 2cx + ab + bc + ca = 0$
 $\Rightarrow 2x^2 - (2a + 2b + 2c)x + (ab + bc + ca) = 0$
(this is a quadratic equation)

In this quadratic equation the discriminate is non-negative. So its roots are real.

Statement - 2 is true and correct explanation of Statement-1.

Ans: (A)

2) **Statement-1** : Equation $(x - a)(x - b) - 2 = 0$, $a < b$ has one root less than a and other root greater than b .,

Ans: ()

Statement-2 : A polynomial equation $f(x) = 0$ has even number of roots between a and b if $f(a)$ and $f(b)$ have opposite signs.

Sol: $(x - a)(x - b) - 2 = 0 \Rightarrow (x - a)(x - b) = 2$
 $\therefore x < a$ and $x > b$ (So, statement-1 is true)
Statement-2 is false

Ans: (C)

3) **Statement-1** : Quadratic equation $f(x) = 0$ has real and distinct roots.

Ans: ()

Statement-2 : quadratic equation $f(x) = 0$ has even number of roots between p and q ($p < q$) if $f(p)$ and $f(q)$ have same sign.

Sol: Quadratic equation $f(x) = 0$; It may have real and imaginary roots.
So, Statement-1 is false.
Statement-2 is true.

Ans: (D)

VI. Very short answer type questions.

1. If $16x^2 - 24x + m = 0$ have equal roots, find the value of m .

Sol: $16x^2 - 24x + m = 0$ have equal roots if $b^2 - 4ac = 0$

$$\Rightarrow (-24)^2 - 4 \times 16 \times m = 0$$

$$\Rightarrow 576 - 64m = 0 \Rightarrow -64m = -576 \Rightarrow m = -576/-64 = 9$$

2. If x_1 and x_2 are the roots of the quadratic equation $ax^2 + bx + c = 0$, find the equation whose roots are $1/x_1$ and $1/x_2$.

Sol: If x_1 and x_2 are the roots of the quadratic equation $ax^2 + bx + c = 0$

$$x_1 + x_2 = -b/a; \quad x_1 x_2 = c/a; \quad \text{Roots are } 1/x_1 \text{ and } 1/x_2$$

$$\therefore \text{the equation is } x^2 - (\alpha + \beta)x + \alpha\beta$$

$$\Rightarrow x^2 - \left(\frac{1}{x_1} + \frac{1}{x_2} \right)x + \frac{1}{x_1} \times \frac{1}{x_2} = 0 \Rightarrow x^2 - \left(\frac{x_2 + x_1}{x_1 x_2} \right)x + \frac{1}{x_1 x_2} = 0$$

$$\Rightarrow x^2 - \left(\frac{-b}{a} \times \frac{a}{c} \right)x + \frac{a}{c} = 0 \Rightarrow x^2 + \frac{bx}{c} + \frac{a}{c} = 0$$

$$\Rightarrow \frac{cx^2 + bx + a}{c} = 0 \Rightarrow cx^2 + bx + a = 0$$

3. Find the value of k , so that $16x^2 + kx + 1 = 0$ has equal roots.

Sol: If $b^2 - 4ac = 0$ then it has equal roots.

$$\Rightarrow k^2 - 4 \times 16 \times 1 = 0 \Rightarrow k^2 = 64 \Rightarrow k = \pm 8$$

4. Find the roots of $x^2 + x - 6 = 0$.

Sol: $x^2 + x - 6 = 0 \Rightarrow x^2 + 3x - 2x - 6 = 0$

$$\Rightarrow x(x+3) - 2(x+3) = 0 \Rightarrow (x-2)(x+3) = 0$$

$$\Rightarrow x - 2 = 0 \quad \text{or} \quad x + 3 = 0$$

$$\Rightarrow x = 2 \quad \text{or} \quad x = -3$$

5. Find the equation whose roots are $3 \pm \sqrt{2}$

Sol: $\alpha = 3 + \sqrt{2}; \quad \beta = 3 - \sqrt{2}$

The equation is $x^2 - (\alpha + \beta)x + \alpha\beta$

$$\Rightarrow x^2 - (3 + \sqrt{2} + 3 - \sqrt{2})x + (3 + \sqrt{2})(3 - \sqrt{2}) = 0 \Rightarrow x^2 - 6x + 7 = 0$$

6. If α and β are the roots of $3x^2 - 5x + 7 = 0$ then find the value of $\alpha^2 + \beta^2$.

Sol: $\alpha + \beta = \frac{-(-5)}{3} = \frac{5}{3}; \quad \alpha\beta = \frac{7}{3}$

$$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$$

$$\Rightarrow \frac{5^2}{3^2} = \alpha^2 + \beta^2 + 2 \times \frac{7}{3} \Rightarrow \alpha^2 + \beta^2 = \frac{14}{3} - \frac{25}{9} = \frac{42 - 25}{9} = \frac{17}{9}$$

7. If α and β are the roots of $ax^2 - ax + b = 0$ then find the value of $\alpha + \beta$ and $\alpha\beta$

Sol: $\alpha + \beta = \frac{-(-a)}{a} = \frac{a}{a} = 1$

$$\alpha\beta = \frac{b}{a}$$

8. The area of a rectangular plot is 528 m². The length of the plot (in metres) is one more than twice its breadth. Write a quadratic equation for the above situation.

Sol: The area of a rectangular plot is 528 m².

The length of the plot (in metres) is one more than twice its breadth.

Let the breadth is x .

$$\therefore \text{Length} = x \times 2 + 1 = 2x + 1$$

$$\Rightarrow x \times (2x + 1) = 528 \Rightarrow 2x^2 + x - 528 = 0$$

The quadratic equation is $2x^2 + x - 528 = 0$

9. Given that $x - \frac{3}{x} = 2$. Write an equivalent quadratic equation for the equation. And find the roots of the equation.

Sol: $x - \frac{3}{x} = 2 \Rightarrow \frac{x^2 - 3}{x} = 2 \Rightarrow x^2 - 3 = 2x \Rightarrow x^2 - 2x - 3 = 0$

An equivalent quadratic equation for the equation $x^2 - 2x - 3 = 0$ is $2x^2 - 4x - 6 = 0$

$$2x^2 - 4x - 6 = 0$$

$$\Rightarrow 2x^2 - 6x + 2x - 6 = 0 \Rightarrow 2x(x - 3) + 2(x - 3) = 0$$

$$\Rightarrow (2x + 2)(x - 3) = 0$$

$$\Rightarrow 2x + 2 = 0 \quad \text{or} \quad x - 3 = 0$$

$$\Rightarrow x = -1 \quad \text{or} \quad x = 3$$

VII. Short Answer Type Questions.

1. Length of a rectangle is 2 units greater than its breadth. If the area of the rectangle is 120 sq. units then find its length.

Sol: Length of a rectangle is 2 units greater than its breadth.

Area of the rectangle is 120 sq. units

Let the breadth = x units; Length = $x + 2$ units

$$\therefore x \times (x + 2) = 120$$

$$\Rightarrow x^2 + 2x - 120 = 0 \Rightarrow x^2 - 10x + 12x - 120 = 0$$

$$\Rightarrow x(x - 10) + 12(x - 10) = 0 \Rightarrow (x + 12)(x - 10) = 0$$

$$\Rightarrow x + 12 = 0 \quad \text{or} \quad \Rightarrow x - 10 = 0$$

$$\Rightarrow x = -12 \quad \text{or} \quad \Rightarrow x = 10$$

2. Is it possible to design a rectangular park of perimeters 60 units and area 200 sq. units. If possible find its length and breadth.

Sol: Perimeter of rectangular park = 60 units.

Let the length of the park = x

$$\therefore 2(x + \text{breadth}) = 60 \Rightarrow x + \text{breadth} = 60/2 = 30 \Rightarrow \text{breadth} = 30 - x$$

Area of the park = 200 sq. Units

$$\Rightarrow \text{length} \times \text{breadth} = 200$$

$$\Rightarrow x \times (30 - x) = 200 \Rightarrow 30x - x^2 - 200 = 0 \Rightarrow x^2 - 30x + 200 = 0$$

$$\Rightarrow x^2 - 10x - 20x + 200 = 0 \Rightarrow x(x - 10) - 20(x - 10) = 0$$

$$\Rightarrow (x - 20)(x - 10) = 0$$

$$\Rightarrow x - 20 = 0 \quad \text{or} \quad x - 10 = 0$$

$$\Rightarrow x = 20 \quad \text{or} \quad x = 10$$

If length of the park is 20 units. Then breadth = $30 - 20 = 10$.

If length of the park is 10 units. Then breadth = $30 - 10 = 20$.

3. The sum of the areas of two squares is 468 m^2 . If the difference of their perimeter is 24 m., then find the sides of the two squares.

Sol: The sum of the areas of two squares is 468 m^2 .

The difference of their perimeter is 24 m.

Let the side of the small square = $x \text{ m}$

Perimeter of small square = $4x \text{ m}$

$$\therefore \text{perimeter of the big square} = 4x + 24$$

$$\Rightarrow \text{big square side} = (4x + 24)/4 = x + 6 \text{ m}$$

$$\therefore \text{area of small square} + \text{area of big square} = 468 \text{ sq. m.}$$

$$\Rightarrow x \times x + (x + 6) \times (x + 6) = 468 \Rightarrow x^2 + x^2 + 12x + 36 = 468$$

$$\Rightarrow 2x^2 + 12x + 36 - 468 = 0 \Rightarrow x^2 + 6x - 216 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times -216}}{2 \times 1} = \frac{-6 \pm \sqrt{900}}{2}$$

$$x = \frac{-6 + 30}{2} \quad \text{or} \quad x = \frac{-6 - 30}{2}$$

$$\Rightarrow x = 12 \quad \text{or} \quad x = -18 \quad (x = -18 \text{ is invalid so } x = 12)$$

Small square side = 12m and big square side = $12 + 6 = 18\text{m}$ (or)

4. Find the dimensions of a rectangle whose perimeter is 12 units and area is 5 square units.

Sol: Rectangle perimeter = 12 units

Area = 5 sq. units.

Let the length = x

$$\therefore 2(\text{length} + \text{width}) = 12 \Rightarrow 2(x + \text{width}) = 12 \Rightarrow x + \text{width} = 12/2 = 6$$

$$\Rightarrow \text{width} = 6 - x$$

$$\therefore \text{length} \times \text{width} = 5$$

$$\Rightarrow x \times (6 - x) = 5 \Rightarrow 6x - x^2 - 5 = 0 \Rightarrow x^2 - 6x + 5 = 0$$

$$\Rightarrow x^2 - x - 5x + 5 = 0 \Rightarrow x(x - 1) - 5(x - 1) = 0$$

$$\Rightarrow x - 5 = 0 \text{ or } x - 1 = 0$$

$$\Rightarrow x = 5 \text{ or } x = 1$$

$$\text{Length} = 5 \text{ and width} = 6 - 5 = 1$$

5. The altitude of a right triangle is 5 m. less than its base. If the hypotenuse is 5 m., find the other two sides.

Sol: Let the base = x m

Altitude = $x - 5$ m

Hypotenuse = 5 m.

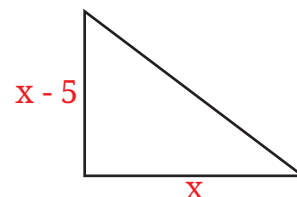
$$\therefore H^2 = P^2 + B^2$$

$$\Rightarrow 5^2 = (x - 5)^2 + x^2 \Rightarrow 25 = x^2 + 25 - 10x + x^2 \Rightarrow 2x^2 - 10x = 0$$

$$\Rightarrow 2x(x - 5) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 5 \text{ (here base} = 0 \text{ is invalid)}$$

$$\text{So base} = 5 \text{ m and altitude} = 5 - 5 = 0$$



6. The sum of the reciprocals of A's age 4 years ago and 4 years from now is $1/3$. Find the present age of A.

Sol: Let A's present age = x years

The sum of the reciprocals of A's age 4 years ago and 4 years from now is $1/3$.

$$\therefore \frac{1}{x - 4} + \frac{1}{x + 4} = \frac{1}{3}$$

$$\Rightarrow \frac{x + 4 + x - 4}{(x - 4)(x + 4)} = \frac{1}{3} \Rightarrow \frac{2x}{x^2 - 16} = \frac{1}{3} \Rightarrow x^2 - 16 = 6x \Rightarrow x^2 - 6x - 16 = 0$$

$$x^2 - 8x + 2x - 16 = 0 \Rightarrow x(x - 8) + 2(x - 8) = 0$$

$$\Rightarrow x = -2 \text{ or } x = 8$$

Age of a person should not be negative. So age of A is 8

7. If a polygon of n sides has $\frac{1}{2}n(n-3)$ diagonals. How many sides will a polygon having 65 diagonals.

Sol: A polygon of n sides has $\frac{1}{2}n(n-3)$ diagonals

Number of diagonals of a polygon is 65

$$\therefore \frac{1}{2}n(n-3) = 65$$

$$\Rightarrow n(n-3) = 130 \Rightarrow n^2 - 3n - 130 = 0$$

$$\Rightarrow n^2 - 13n + 10n - 130 = 0 \Rightarrow n(n-13) + 10(n-13) = 0$$

$$\Rightarrow n = -10 \text{ or } n = 13 \text{ (Here } n = -10 \text{ is invalid)}$$

So number of sides of the polygon = 13

8. A two digit number is such that the product of its digits is 15. If 18 is added to the number, the digits interchange their places. Find the number.

Sol: A two digit number is such that the product of its digits is 15

Let the unit digit = x

$$\therefore \text{Tens digit} = 15/x$$

$$\therefore \text{The number} = \frac{15}{x} \times 10 + x = \frac{150}{x} + x = \frac{150 + x^2}{x}$$

$$\text{After interchange the digits, the number} = x \times 10 + \frac{15}{x} = 10x + \frac{15}{x} = \frac{10x^2 + 15}{x}$$

If 18 is added to the number, the digits interchange their places.

$$\frac{150 + x^2}{x} + 18 = \frac{10x^2 + 15}{x}$$

$$\Rightarrow \frac{150 + x^2 + 18x}{x} = \frac{10x^2 + 15}{x}$$

$$\Rightarrow 150 + x^2 + 18x = 10x^2 + 15 \Rightarrow 150 + x^2 + 18x - 10x^2 - 15 = 0$$

$$\Rightarrow -9x^2 + 18x + 135 = 0 \Rightarrow 9x^2 - 18x - 135 = 0$$

$$\Rightarrow x^2 - 2x - 15 = 0 \Rightarrow x^2 - 5x + 3x - 15 = 0$$

$$\Rightarrow x(x-5) + 3(x-5) = 0$$

$$\Rightarrow x = -3 \text{ or } x = 5$$

If the unit digit is -3 then the tens digit = $15/-3 = -5$

If the unit digit is 5 then the tens digit = $15/5 = 3$

9. If -4 is a root of the quadratic equation $x^2 + px - 4 = 0$ and the roots of $x^2 + px + k = 0$ has equal roots, then find the value of k .

Sol: -4 is a root of the quadratic equation $x^2 + px - 4 = 0$

$$\therefore (-4)^2 + p(-4) - 4 = 0$$

$$\Rightarrow 16 - 4p - 4 = 0$$

$$\Rightarrow -4p = -12 \Rightarrow p = -12/-4 = 3$$

$$x^2 + px + k = 0$$

$$\Rightarrow x^2 + 3x + k = 0$$

$$\text{Equal roots if } b^2 - 4ac = 0$$

$$\Rightarrow 3^2 - 4 \times 1 \times k = 0 \Rightarrow 9 - 4k = 0 \Rightarrow -4k = -9 \Rightarrow k = 9/4$$

10. The product of two consecutive positive integers is 650. Find the integers.

Sol: Let the two consecutive positive integers are x and $x+1$.

$$\therefore x(x+1) = 650$$

$$\Rightarrow x^2 + x - 650 = 0 \Rightarrow x^2 + 26x - 25x - 650 = 0$$

$$\Rightarrow x(x+26) - 25(x+26) = 0$$

$$\Rightarrow x = 25 \text{ or } x = -26 \quad (x = -26 \text{ is not valid})$$

Two numbers are 25 and 26

VIII. Long answer type Questions.

1. The base of a triangle is 4cm longer than its altitude. If the area of the triangle is 48 sq.cm then find its base and altitude.

Sol: Let the altitude = x cm

$$\therefore \text{Base} = x + 4 \text{ cm}$$

Area of the triangle is 48 sq.cm

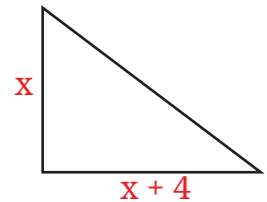
$$\therefore \frac{1}{2} \times \text{Base} \times \text{altitude} = 48 \Rightarrow \frac{1}{2} \times (x+4) \times x = 48$$

$$\Rightarrow (x+4) \times x = 48 \times 2 = 96 \Rightarrow x^2 + 4x - 96 = 0$$

$$\Rightarrow x^2 + 12x - 8x - 96 = 0 \Rightarrow x(x+12) - 8(x+12) = 0$$

$$\Rightarrow x = 8 \text{ or } x = -12 \quad (x = -12 \text{ invalid, so } x = 8)$$

Altitude = 8 cm and base = $8 + 4 = 12$ cm.



2. The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Sol: Let the shorter side of the rectangle = x m

$$\text{Diagonal} = x + 60 \text{ m}$$

$$\text{Longer side of the rectangle} = x + 30$$

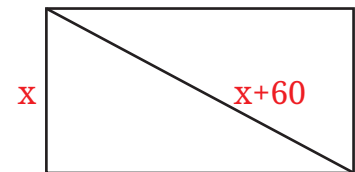
$$\therefore x^2 + (x+30)^2 = (x+60)^2$$

$$\Rightarrow x^2 + x^2 + 900 + 60x = x^2 + 3600 + 120x$$

$$\Rightarrow x^2 - 60x - 2700 = 0 \Rightarrow (x+30)(x-90) = 0$$

$$x = -30 \text{ or } x = 90 \quad (x = -30 \text{ invalid, so } x = 90)$$

The shorter side = 90m And the longer side = 120 m



3. A motor boat heads upstream a distance of 24 km on a river whose current is running at 3 km per hour. The trip up and back takes 6 hours. Assuming that the motor boat maintained a constant speed, what was its speed?

Sol: Let the speed of the boat = x km/hour

The speed of the current = 3 km/h.

The speed of the boat upstream = $(x - 3)$ km/h

The speed of the boat down stream = $(x + 3)$ km/h.

$$\therefore \frac{24}{x-3} + \frac{24}{x+3} = 6$$

$$\Rightarrow 24x + 72 + 24x - 72 = 6x^2 - 54$$

$$\Rightarrow 6x^2 - 48x - 54 = 0 \Rightarrow 6(x^2 - 8x - 9) = 0 \Rightarrow (x - 9)(x + 1) = 0$$

So $x = 9$ km/h

The speed of the boat is 9 km./h.

4. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Sol: Let the speed of the train = x

The time taken to go 360 km = $\frac{360}{x}$ h.

If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey.

$$\therefore \frac{360}{x} - 1 = \frac{360}{x+5} \Rightarrow \frac{360-x}{x} = \frac{360}{x+5}$$

$$\Rightarrow 360x - x^2 + 1800 - 5x = 360x$$

$$\Rightarrow x^2 + 5x - 1800 = 0$$

$$\Rightarrow (x + 45)(x - 40) = 0$$

$x = -45$ or $x = 40$ ($x = -45$ is invalid, so $x = 40$)

The speed of the train is 40 km/h

5. Show that the product of the roots of a quadratic equation is ' c/a '

Sol: The product of the roots of a quadratic equation

$$= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \times \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{(-b)^2 - b^2 + 4ac}{4a^2} = \frac{c}{a}$$

6. Two water taps together can fill a tank in 12 hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Sol: Let the tank fills by the small tap in x hours

In one hour it will fill = $\frac{1}{x}$ part

The big tap fills the tank in $x - 10$ hours

In one hour it fills $\frac{1}{x-10}$ part

$$\therefore \frac{1}{\frac{1}{x-10} + \frac{1}{x}} = 12 \Rightarrow \frac{1}{\frac{x+x-10}{x(x-10)}} = 12 \Rightarrow \frac{x(x-10)}{x+x-10} = 12$$

$$\Rightarrow x^2 - 10x = 24x - 120 \Rightarrow x^2 - 34x + 120 = 0 \Rightarrow (x-30)(x-4) = 0$$

So $x = 30$ or 4

The smaller tap fills the tank in 30 hours and the bigger tap fills the tank in 20 hours.

7. A two digit number is such that the product of the digits is 8. When 18 is added to the number they interchange their places. Determine the number.

Sol: A two digit number is such that the product of the digits is 8.

Let the tens digit = x

Ones digit = $8/x$

When 18 is added to the number they interchange their places.

$$\therefore 10x + \frac{8}{x} + 18 = 10 \times \frac{8}{x} + x \Rightarrow \frac{10x^2 + 8 + 18x}{x} = \frac{80 + x^2}{x}$$

$$\Rightarrow 9x^2 + 18x - 72 = 0$$

$$\Rightarrow 9(x-4)(x+2) = 0$$

$$\Rightarrow x = 4$$

The tens digit is 4 and ones digit is 2.

8. Sum of the areas of two squares is 468 m^2 . If the difference of their perimeters is 24 m, find the sides of the two squares.

Sol: See VII - Question 3 - Answer

9. Find two numbers whose sum is 27 and product is 182.

Sol: Two numbers whose sum is 27 and product is 182.

Let first number = x

Second number = $27 - x$ (or) $182/x$

$$\therefore 27 - x = \frac{182}{x} \Rightarrow x(27 - x) = 182$$

$$\Rightarrow -x^2 + 27x - 182 = 0 \Rightarrow x^2 - 27x + 182 = 0$$

$$\Rightarrow x^2 - 13x - 14x + 182 = 0 \Rightarrow x(x - 13) - 14(x - 13) = 0$$

$$\Rightarrow x = 14 \text{ (or) } x = 13$$

If the first number = 14 then the second number = $27 - 14 = 13$

If the first number = 13 then the second number = $27 - 13 = 14$

10. If the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - a - ac = 0$ are equal, prove that either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$.

Sol: $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$$\Rightarrow a = (c^2 - ab), b = -2(a^2 - bc), c = (b^2 - ac)$$

Roots of the quadratic eqn. are equal

$$\Rightarrow d = 0$$

$$\Rightarrow b^2 - 4ac = 0$$

$$\Rightarrow b^2 - 4ac = 0$$

$$\Rightarrow b^2 = 4ac$$

$$\Rightarrow [-2(a^2 - bc)]^2 = 4(c^2 - ab)(b^2 - ac)$$

$$\Rightarrow 4(a^2 - bc)^2 = 4(c^2 - ab)(b^2 - ac)$$

$$\Rightarrow a^4 + b^2c^2 - 2a^2bc = b^2c^2 - ac^3 - ab^3 + a^2bc$$

$$\Rightarrow a^4 + ab^3 + ac^3 = 3a^2bc.$$

$$\Rightarrow a(a^3 + b^3 + c^3 - 3abc) = 0.$$

$$\text{So } a = 0 \text{ or } a^3 + b^3 + c^3 = 3abc$$

11. If a root of $px^2 + qx + r = 0$ is thrice the other root, then show that $3q^2 = 16pr$

Sol: If a root of $px^2 + qx + r = 0$ is thrice the other root

Let α is a roots of given equation

$$\therefore \text{other root} = 3\alpha$$

$$\text{Sum of roots } \alpha + 3\alpha = -q/p \Rightarrow 4\alpha = -q/p \Rightarrow \alpha = -q/4p$$

$$\text{Products of roots } \alpha \times 3\alpha = r/p \Rightarrow 3\alpha^2 = r/p \text{ (replacing } \alpha \text{ by } -q/4p)$$

$$\Rightarrow 3 \times \left(\frac{-q}{4p} \right)^2 = \frac{r}{p} \Rightarrow \frac{3q^2}{16p^2} = \frac{r}{p}$$

$$\Rightarrow 3q^2 = 16pr$$

GOLD PACKAGE



I. Choose the correct answer.

- 1) If the equation $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$ and $a, b, c \in \mathbb{R}$, have a common root, then $a:b:c$ is
- a) 1:2:3 b) 3:2:1 c) 1:3:2 d) None of these

Ans: (A) 1:2:3

- 2) If α and β are the distinct roots of the equation $x^2 - x + 1 = 0$, the $\alpha^{101} + \beta^{107}$ is equal to :
- a) 0 b) 1 c) 2 d) None of these

Sol: $x^2 - x + 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 1 \times 1}}{2 \times 1}$$

$$\Rightarrow x = \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm i\sqrt{3}}{2}$$

$$\alpha = \frac{1}{2} + \frac{i\sqrt{3}}{2} = -\omega^2$$

$$\beta = \frac{1}{2} - \frac{i\sqrt{3}}{2} = -\omega$$

$$\begin{aligned} \alpha^{101} + \beta^{107} &= -(\omega^{202} + \omega^{107}) = -(\omega^{201} \times \omega + \omega^{105} \times \omega^2) \\ &= -(\omega + \omega^2) = -(-1) = 1 \end{aligned}$$

$1, \omega, \omega^2 \Rightarrow$ Cube roots of unity

$$\omega = -\frac{1}{2} + \frac{i\sqrt{3}}{2} = -\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)$$

$$\omega^2 = -\frac{1}{2} - \frac{i\sqrt{3}}{2} = -\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)$$

$$\omega^{3n} = 1$$

$$1 + \omega + \omega^2 = 0$$

$$\Rightarrow \omega + \omega^2 = -1$$

Ans: (B) 1

- 3) The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has:
- a) exactly four real roots b) no real roots c) infinite real roots d) None of these

Sol: $e^{\sin x} - e^{-\sin x} - 4 = 0$

$$\text{Let } e^{\sin x} = t$$

$$\Rightarrow t - \frac{1}{t} - 4 = 0 \Rightarrow t^2 - 4t - 1 = 0$$

$$\Rightarrow t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times -1}}{2 \times 1} = \frac{16 \pm \sqrt{20}}{2} = 2 \pm \sqrt{5}$$

$$\Rightarrow t = 2 + \sqrt{5}, 2 - \sqrt{5}$$

But t is a real positive number

$$\therefore t = 2 + \sqrt{5} \Rightarrow e^{\sin x} = 2 + \sqrt{5} \Rightarrow \sin x = \log_e(2 + \sqrt{5})$$

$$\text{but } 2 + \sqrt{5} > e \Rightarrow \log_2(2 + \sqrt{5}) > \log_e e$$

$$\therefore \log_e(2 + \sqrt{5}) > 1$$

Hence, $\sin x > 1$ (which is not possible) So, the given equation has no real solution.

Ans: (B) No real roots

- 4) Product of real roots of the equation $t^2x^2 + |x| + 9 = 0$
 a) is always positive b) is always negative c) does not exist d) None of these

Sol: Here in the above question we have equation $t^2x^2 + |x| + 9 = 0$

$$t^2x^2 \geq 0, |x| \geq 0 \text{ and } 9 \geq 0$$

So, on adding these inequalities we get $t^2x^2 + |x| + 9 \geq 0$ as long as t and x are assumed real

Hence no real roots exist.

Ans: (C) Does not exist

- 5) If p and q are the roots of the equation $x^2 + px + q = 0$, then
 a) $p = 1, q = -2$ b) $p = 0, q = 1$ c) $p = -2, q = 0$ d) None of these

Sol: Given equation $x^2 + px + q = 0$

Also, given p, q are the roots of the equation.

$$p + q = -p \Rightarrow 2p = -q \dots\dots\dots(1)$$

$$pq = q \Rightarrow pq - q = 0 \Rightarrow q(p - 1) = 0$$

$$\Rightarrow q = 0 \text{ or } p = 1$$

$$\text{So, by equation (1), } q = 0 \Rightarrow p = 0$$

$$\text{So, by equation (1), } p = 1 \Rightarrow q = -2$$

Hence, the values of p and q are $0, 0$ or $1, -2$

Ans: (A) $p = 1, q = -2$

- 6) The number of real solutions of the equation $x^2 - 3x + 2 = 0$ is:
 a) 2 b) 4 c) 1 d) None of these

Sol: $x^2 - 3x + 2 = 0 \Rightarrow x^2 - 2x - x + 2 = 0$

$$\Rightarrow x(x - 2) - 1(x - 2) = 0$$

$$\Rightarrow (x - 1)(x - 2) = 0$$

$$\Rightarrow x = \pm 1 \text{ or } x = \pm 2$$

Ans: (B) 4

- 7) If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation having α and β as its roots is:
 a) $3x^2 - 19x + 3 = 0$ b) $3x^2 + 19x - 3 = 0$ c) $3x^2 - 19x - 3 = 0$ d) None of these

Sol: Given $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$

$$\alpha^2 = 5\alpha - 3 \Rightarrow \alpha^2 - 5\alpha + 3 = 0$$

$$\Rightarrow \alpha = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times 3}}{2 \times 1} = \frac{5 \pm \sqrt{25 - 12}}{2} = \frac{5 \pm \sqrt{13}}{2} = \frac{5}{2} \pm \frac{\sqrt{13}}{2}$$

$$\beta^2 = 5\beta - 3 \Rightarrow \beta^2 - 5\beta + 3 = 0$$

$$\Rightarrow \beta = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times 3}}{2 \times 1} = \frac{5 \pm \sqrt{25 - 12}}{2} = \frac{5 \pm \sqrt{13}}{2} = \frac{5}{2} \pm \frac{\sqrt{13}}{2}$$

The equation = $x^2 - x(\alpha + \beta) + \alpha\beta = 0$

$$\Rightarrow x^2 - x\left(\frac{5}{2} + \frac{\sqrt{13}}{2} + \frac{5}{2} - \frac{\sqrt{13}}{2}\right) + \left(\frac{5}{2} + \frac{\sqrt{13}}{2}\right)\left(\frac{5}{2} - \frac{\sqrt{13}}{2}\right) = 0$$

$$\Rightarrow x^2 - 2x + \frac{25}{4} - \frac{5\sqrt{13}}{4} + \frac{5\sqrt{13}}{4} - \frac{13}{4} = 0$$

$$\Rightarrow x^2 - 2x + \frac{38}{4} = 0$$

$$\Rightarrow \frac{4x^2 - 8x + 38}{4} = 0 \Rightarrow 4x^2 - 8x + 38 = 0$$

Ans: (D) None of these

8) Which of the following is a pure quadratic equation?

a) $ax^2 - bx - c = 0$

b) $ax^2 + bx + c = 0$

c) $ax^2 + c = 0$

d) None of these

Sol: The pure quadratic equation is in the form of $ax^2 + c = 0$ where no linear term i.e. x^1 term exist.

Ans: (C) $ax^2 + c = 0$

II. Answer the following questions.

1) If the equation $(k-2)x^2 - (k-4)x - 2 = 0$ has difference of roots as 3 then find the value of k.

Sol: $(k-2)x^2 - (k-4)x - 2 = 0$

$$\alpha + \beta = \frac{k-4}{k-2}$$

$$\alpha\beta = \frac{-2}{k-2}$$

$$\alpha - \beta = 3$$

$$\Rightarrow \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} = 3 \Rightarrow \sqrt{\left(\frac{k-4}{k-2}\right)^2 - 4\left(\frac{-2}{k-2}\right)} = 3 \Rightarrow \frac{k}{k-2} = 3$$

$$\Rightarrow k = 3k - 6$$

$$\therefore k = 3$$

2) If α, β are the roots of a quadratic equation $x^2 - 3x + 5 = 0$ then write an equation whose roots are $(\alpha^2 - 3\alpha + 7)$ and $(\beta^2 - 3\beta + 7)$.

Sol: $x^2 - 3x + 5 = 0$

So $\alpha^2 - 3\alpha + 5 = 0$

$$\Rightarrow \alpha^2 - 3\alpha + 7 = 2$$

Same like this

$$\beta^2 - 3\beta + 7 = 2$$

$\therefore 2$ and 2 are roots of the equation So the equation is $x^2 - 4x + 4 = 0$

- 3) If α, β are the roots of a quadratic equation $x^2 - 5x + 6 = 0$ then write an equation whose roots are $\alpha + 3$ and $\beta + 3$.

Sol: $\alpha + 3 = x$ so $\alpha = x - 3$

So the required equation is $(x - 3)^2 - 5(x - 3) + 6 = 0$

$$\Rightarrow x^2 - 11x + 30 = 0$$

- 4) If one root of $x^2 - 5x + k = 0$ is 2, then find the value of k and the other root.

Sol: One root of the equation $x^2 - 5x + 4 = 0$ is 2

$$\text{Let } \beta = 2; \quad \alpha + \beta = 5 \Rightarrow \alpha = 3$$

$$k = \alpha \times \beta = 2 \times 3 = 6$$

- 5) The sides of an equilateral triangle are shortened by 12 units, 13 units and 14 units respectively and a right angle triangle is formed. Find the length of each side of the equilateral triangle.

Sol: Let the sides of the equilateral triangle is x

The sides of the right angled triangle $x - 12, x - 13$ and $x - 14$

$$\therefore (x - 12)^2 = (x - 13)^2 + (x - 14)^2$$

$$\Rightarrow x^2 + 144 - 24x = x^2 + 169 - 26x + x^2 + 196 - 28x$$

$$\Rightarrow x^2 - 30x + 221 = 0$$

$$\Rightarrow (x - 17)(x - 13) = 0$$

$$\Rightarrow x = 17 \text{ or } x = 13$$

The sides of the equilateral triangle is 17. (because 13 is not possible)

- 7) If $x = \dots$ then find the value of x .

Sol:

- 8) If α, β are the roots of $2x^2 + x + 3 = 0$, then find the equation whose roots are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$.

Sol: $2x^2 + x + 3 = 0$

$$\alpha + \beta = \frac{-1}{2} \quad \text{and} \quad \alpha\beta = \frac{3}{2}$$

$$\frac{1-\alpha}{1+\alpha} + \frac{1-\beta}{1+\beta} = \frac{2(1-\alpha\beta)}{1+\alpha+\beta+\alpha\beta} = \frac{2\left(1-\frac{3}{2}\right)}{1+\frac{-1}{2}+\frac{3}{2}} = \frac{-1}{2}$$

$$\frac{1-\alpha}{1+\alpha} \times \frac{1-\beta}{1+\beta} = \frac{(1-(\alpha+\beta)+\alpha\beta)}{1+\alpha+\beta+\alpha\beta} = \frac{1-\frac{-1}{2}+\frac{3}{2}}{2} = \frac{3}{2}$$

Τη εθυατιον ισ $2x^2 + x + 3 = 0$